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SYNTHESIS OF THE CONTROL UNIT OF A SERVOSYSTEM OPTIMUM FROM THE POINT OF VIEW OF SPEED OF RESPONSE

Sung Chien

(Moscow)

The switching surface for a servosystem which is optimum from the point of view of speed of response is determined. The system possesses second-order astatism and is described by a third-order differential equation. Approximations to this surface are carried out, and a control unit which provides almost optimum transient processes in the system is designed. Results of the experimental investigation of a system with an electronic model of the member are given.

INTRODUCTION

Many papers have been devoted to the theory of regulation and control systems optimum from the point of view of their high speed. The theory for the synthesis of systems optimum from the point of view of their high speed has been developed for a certain class of controlled members, thus, permitting the synthesis of the control unit of an optimum system [1-8]. However, the system which is obtained on the basis of such an accurate synthesis usually has a very complex circuit and contains a large number of components which serve to carry out the computational operations. Consequently, in practice it is usually not expedient to achieve a strictly optimum system, the given basic part of which is described by a third- and higher-order differential equation.

The article is devoted to the examination of a method of synthesizing a system close to an optimum one, enabling one to design a relatively simple-control unit providing almost optimum transient processes. These processes are practically the same as those in strictly optimum ones for a certain class of external effects. The method presented is based on the approximation of the switching hypersurface by a simpler surface. The approximation is achieved by expanding functions of many variables into sums and products of functions of one variable. A servosystem whose power unit is described by a third-order differential equation and possesses second-order astatism is examined as an example. It is assumed that a restriction is imposed on the modulus of the control signal fed to the input of the system's power unit.

It is necessary to design for this system a control unit which ensures optimum transient processes for all permissible input signals belonging to the class of linear functions of the form

$$X_0(t) = A_0 + A_1 t, \tag{1}$$

where A_0, and A_1 are constant coefficients, one of which can in a particular case equal zero.

1. Construction of the Switching Surface in Three-Dimensional Phase Space

Let us assume that the system's power unit is of the form shown as a block schematic in Fig. 1, and is described by the differential equation

$$T\frac{d^3X}{dt^3} + \frac{d^2X}{dt^2} = k_0 v(t), \tag{2}$$

where X(t) is the output quantity, v(t) is the control signal, and k_0 is the amplification factor of the system's power unit.

The restriction imposed on the system is of the form:

$$|k_0 v(t)| \leqslant M, \tag{3}$$

where M is some constant quantity.

Fig. 1

In accordance with the theorem which was proved in paper [2] for the case when all the roots of the characteristic equation are real, the optimum process consists of three regions, where each region is described by a differential equation of the form

$$T\frac{d^3X}{dt^3} + \frac{d^2X}{dt^2} = \sigma M, \tag{4}$$

where $\sigma = \pm 1$ alternately in adjacent regions, $M = k_0 v_0$, while v_0 is the maximum permissible value of $|v(t)|_{max}$.

Let us first assume that the input signal is a permissible one and varies according to the square law

$$X_0(t) = A_0 + A_1 t + A_2 t^2. \tag{5}$$

Let us use x' to denote the error

$$x' = X_0(t) - X(t). \tag{6}$$

Replacing X by $X_0 - x'$ in (4), we obtain the differential equation with respect to the error

$$T\frac{d^3x'}{dt^3} + \frac{d^2x'}{dt^2} = 2A_2 - \sigma M. \tag{7}$$

Let us reduce all the variables in Eq. (7) to relative units. For this we introduce the notations

$$t = Tt', \quad x = \alpha x, \quad \alpha = (2A_2 + M)T^2, \tag{8}$$

Then Eq. (7) can be rewritten in the form

$$\frac{d^3x}{dt'^3} + \frac{d^2x}{dt'^2} = \frac{(2A_2 - \sigma M)T^2}{\alpha}. \tag{9}$$

From Eq. (9) it can be seen that the differential equation contains the parameter of the input effect A_2. In this case the switching surface in phase space is quasi-stationary, i.e., depending on the nature of the input effect.

In order to construct the surface S we reverse the direction of the time count, and choose the initial time to be at the instant t_0 corresponding to the end of the transient process. Introducing $\tau = t_0/T - t'$, we obtain

$$\frac{d^3x}{d\tau^3} - \frac{d^2x}{d\tau^2} = \frac{(\sigma M - 2A_2)T^2}{\alpha} = \eta, \tag{10}$$

where

$$\eta = \begin{cases} -1 & \text{for} \quad \sigma = -1, \\ \dfrac{M - 2A_2}{M + 2A_2} = \theta \ (0 \leqslant \theta \leqslant 1) & \text{for} \quad \sigma = +1. \end{cases}$$

These values are obtained if α is substituted from (8).

Solving differential Eq. (10) with initial conditions $x(0) = 0$, $\left(\frac{dx}{d\tau}\right)_0 = 0$, $\left(\frac{d^2x}{d\tau^2}\right)_0 = 0$, we obtain a system

of parametric equations determining the line L_1 on which lie the last regions of all the optimum trajectories:

$$x = \eta_1 (1 + \tau - e^{\tau} + \tfrac{1}{2}\tau^2),$$
$$y = \frac{dx}{dt'} = -\frac{dx}{d\tau} = \eta_1 (e^{\tau} - \tau - 1), \tag{11}$$
$$z = \frac{d^2x}{dt'^2} = \frac{dx}{d\tau^2} = \eta_1 (1 - e^{\tau}).$$

The quantity η_1 is a certain value of η (see below).

The next to the last segments of the set of optimum trajectories ending on line L_1 form the surface L_2 which contains L_1. As it has been shown in [6], this surface is the switching surface S which divides all the space into two half-spaces corresponding to the values $\sigma = +1$ and $-\sigma = -1$. Surface L_2 or S can be determined from the general solution of differential Eq. (10) with initial conditions

$$x_{10} = \eta_1 \left(1 + \tau_{10} - e^{\tau_{10}} + \frac{1}{2}\tau_{10}^2\right),$$
$$y_{10} = \eta_1 (e^{\tau_{10}} - \tau_{10} - 1), \tag{12}$$
$$z_{10} = \eta_1 (1 - e^{\tau_{10}}),$$

where τ_{10} is a fixed value of τ determining the point (x_{10}, y_{10}, z_{10}) on line L_1. Taking conditions (12) into account we solve Eq. (10) and obtain a system of parametric equations for surface S:

$$x = \eta_1 \left(\tau_{10} + \frac{1}{2}\tau_{10}^2 + \frac{\eta_2}{\eta_1}\right) + \eta_1 \left(\tau_{10} + \frac{\eta_2}{\eta_1}\right)\tau + \eta_1 \left(1 - e^{\tau_{10}} - \frac{\eta_2}{\eta_1}\right)e^{\tau} + \frac{\eta_2}{2}\tau^2,$$
$$y = \eta_1 \left(-\tau_{10} - \frac{\eta_2}{\eta_1}\right) - \eta_1 \left(1 - e^{\tau_{10}} - \frac{\eta_2}{\eta_1}\right)e^{\tau} - \eta_2\tau, \tag{13}$$
$$z = \eta_1 \left(1 - e^{\tau_{10}} - \frac{\eta_2}{\eta_1}\right)e^{\tau} + \eta_2.$$

Here η_1 denotes the controlling effect on the last interval of the trajectory, and η_2 denotes the same on the next to last interval. Let us note that η_1 and η_2 always have opposite signs. The quantities η_1 and η_2 can take on the values -1 or θ, depending on which half-space the characteristic point is located in at time $t = 0$. In this way, the system of equations in (13) determines the surface S. The line L_1 divides S into two half-surfaces. One of them, L'_2, is obtained from (13) if we set $\eta_1 = -1$ and $\eta_2 = \theta$; the other, L''_2, is obtained when $\eta_1 = \theta$ and $\eta_2 = 1$.

The problem of synthesizing the optimum control section of a system reduces to the formation of a switching plane in phase space with equation

$$\phi(x, y, z) = 0 \tag{14}$$

The function $\phi(x, y, z)$ must possess the following properties: for all points in space located above the surface S, the inequality

$$\phi(x, y, z) > 0 \tag{15}$$

holds, while for points below the surface S, the inequality

$$\varphi(x, y, z) < 0. \tag{16}$$

holds.

Such a function ϕ can serve as the control function in an optimum system. Every device which can generate a function satisfying conditions (14), (15), and (16) can serve as the control section of the optimum system. In view of the transcendental nature of Eqs. (13) it is, in general, impossible to exclude parameters τ and τ_{10} from them. They can be eliminated by means of an automatic computing device. Moreover, it is necessary to have in the system a device to continuously measure the parameter A_2 which appears in Eq. (13). All this makes the system extremely complex.

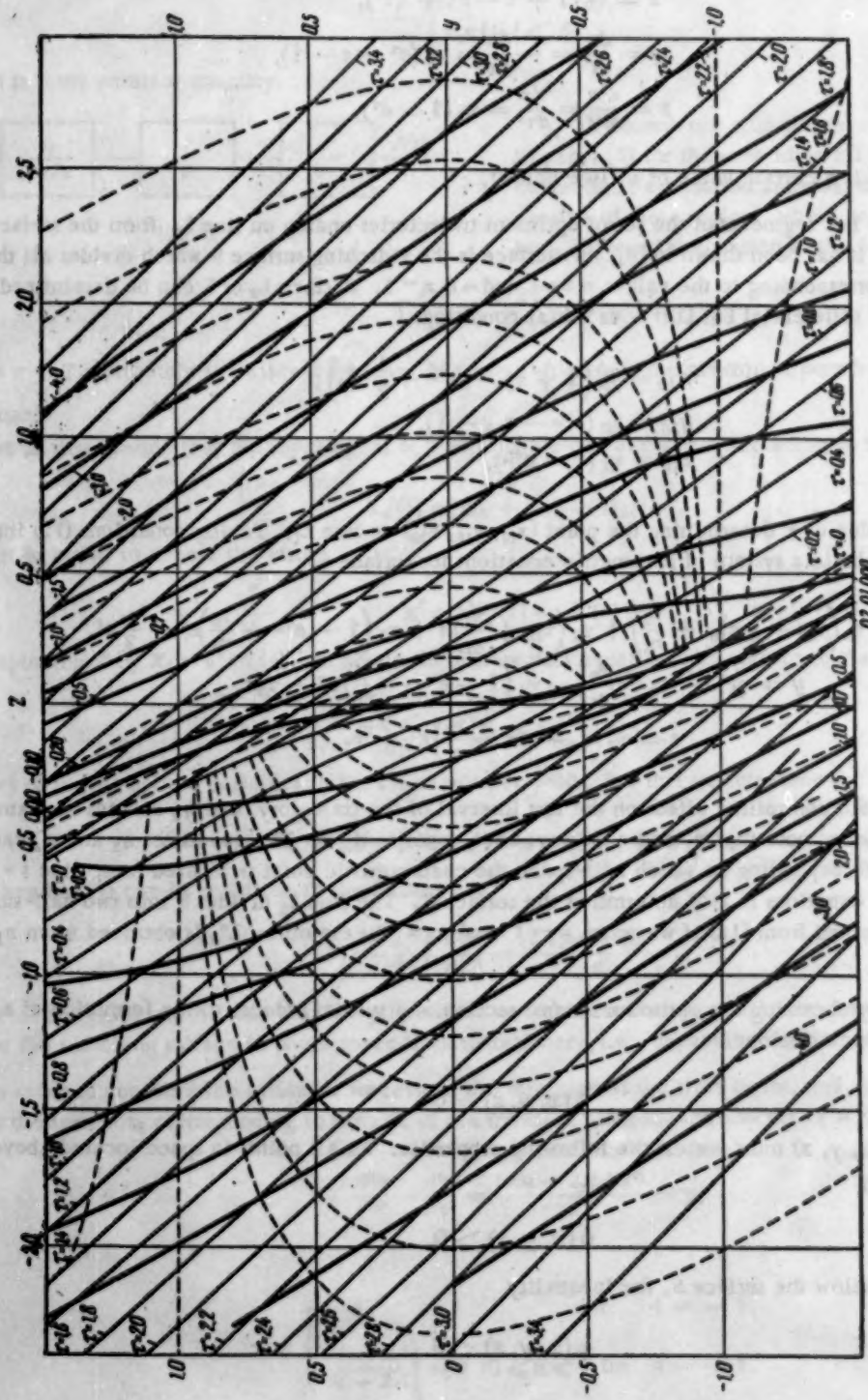

Fig. 2

However, if the input effect has the form of (1), i.e., $A_2 = 0$, surface S becomes independent of the parameters of the input effect. In Eqs. (13) η_1 and η_2 will only take on values of +1 or −1. In this case Eqs. (13) simplify and take on the form

$$x = \eta_{i1}\left(\tau_{10} + \frac{1}{2}\tau_{10}^2 - 1\right) + \eta_{i1}(\tau_{10} - 1)\tau + \eta_{i1}(2 - e^{\tau_{10}})e^{\tau} - \frac{\eta_{i1}}{2}\tau^2,$$
$$y = \eta_{i1}(1 - \tau_{10}) - \eta_{i1}(2 - e^{\tau_{10}})e^{\tau} + \eta_{i1}\tau, \qquad (17)$$
$$z = \eta_{i1}(2 - e^{\tau_{10}})e^{\tau} - \eta_{i1}.$$

When $\eta_1 = -1$, Eqs. (17) give the half-surface L'_2, while when $\eta_1 = +1$ they give the half-surface L''_2. In this case the surface S is antisymmetric with respect to the origin of coordinates. To construct surface S it is sufficient to construct one half, say L'_2, for $\eta_1 = -1$, and the other half is obtained by an antisymmetric reflection with respect to the origin.

In view of the transcendental nature of Eq. (17) with respect to parameters τ and τ_{10}, Eq. (14) cannot be considered. As a result, it is impossible to construct an optimum system without complex computing devices with feedback loops. The projections of surface S, defined by the system of equations in (17), onto the (y, z) plane are shown in Fig. 2, where the heavy lines connect equal values of $f(y, z)$, the thin lines are isochrons, and the broken lines are the optimum trajectories, each of which corresponds to a particular value of τ_{10}. The right (relative to the isochron $\tau = 0$) half-surface corresponds to L'_2, the left half-surface corresponds to L''_2. As it was pointed out at the beginning of this paper, we will henceforth conduct the synthesis of an optimum system specifically for the latter case when $A_2 = 0$.

2. Approximation of the Surface S and the Synthesis of the Optimum Control Unit

The object of the approximation is to replace the surface S by a simpler surface S* which is convenient for the synthesis of the control unit. The approximating function must possess rapidly converging approximations and the possibility of obtaining any degree of accuracy of the approximation. The nature of the approximating function must permit its reproduction by means of relatively simple equipment.

As a criterion of the approximation of S by S* let us take the minimum mean-square difference, i.e., the minimum of the integral

$$I = \int_a^b \int_c^d [f(y, z) - f^*(y, z)]^2 \, dy \, dz, \qquad (18)$$

where $f(y, z)$ is the approximated function, $f^*(y, z)$ is the approximating function, a, b, c, d are numbers corresponding to the boundaries of the region over which the function is specified.

The condition for a minimum in I is satisfied by the two-row Fourier series

$$f^*(y, z) = \sum_{k,l} c_{kl} g_k(y) h_l(z), \qquad (19)$$

where g_k and h_l are a system of orthonormal functions, while c_{kl} is the coefficient of the Fourier expansion.

For the case when the function $f(y, z)$ is specified in the form of a graph or table, there is a numerical method of expanding into an initially unknown system of linearly independent functions of one variable, and for $f^*(y, z)$ one can take the final sum of the terms in the expansions [9, 10]. In this case, instead of (19), we can write

$$f^*(y, z) = g_0(y) + h_0(z) + \sum_{i=0}^{n} g_i(y) h_i(z). \qquad (20)$$

Here $g_i(y)$ and $h_i(z)$ are functions of one variable. They are determined from the condition for a minimum of the integral in (18). As I. I. Éterman showed in [10], g_i and h_i satisfy Friedholm's homogeneous integral equation of the second kind and are eigenfunctions of this equation. Hence, series (20) is orthogonal. Experience shows that series (20) usually converges rapidly. It satisfies the requirements imposed on the approximating function.

Expression (20) shows that to construct the control unit of an optimum system it is necessary to have only a small number of nonlinear functional converters and multiplying and summing units. The accuracy of the solution of the problem can be increased by attaching more units corresponding to the subsequent terms in the expansion of (20), without changing the parameters of the earlier units. Subsequently, the expansion of (20) will serve as a basis for the solution of the given synthesis problem.

Firstly, we will establish the basic region over which surface S is specified. We will show that for the synthesis of an optimum system it is sufficient to limit the values of the variables $\underline{y}$ and $\underline{z}$ to within the limits of a unit square ($-1 \leq y \leq 1$, $-1 \leq z \leq 1$). For this we must find the maximum possible values y_{max} and z_{max}.

As it can be seen from Fig. 1, after a constant controlling effect is delivered to the input of the power unit, the output takes on the following steady-state value:

$$\lim_{t\to\infty}\left(\frac{d^2X}{dt^2}\right)=\left(\frac{d^2X}{dt^2}\right)_{max}=\lim_{p\to 0}\left(\frac{k_0v_0}{1+Tp}\right)=k_0v_0.$$

From here, taking (8) into account, we have

$$|z|_{max}=\left|\frac{d^2x}{dt'^2}\right|_{max}=\frac{T^2}{\alpha}\left|\frac{d^2x}{dt^2}\right|_{max}=1. \tag{21}$$

Let us examine the maximum value of $\underline{y}$, i.e., y_{max}:

$$|y|_{max}=\left|\frac{dx}{dt'}\right|_{max}=\frac{T}{\alpha}\left|A_1-\frac{dX}{dt}\right|_{max}\leqslant\left(|A_1|+\left|\frac{dX}{dt}\right|_{max}\right)\frac{T}{\alpha}.$$

On the other hand, from (7), when $A_2=0$, $\sigma=-1$, and zero initial conditions, we find

$$\frac{dX}{dt}=k_0v_0\left[t+T\left(e^{-\frac{t}{T}}-1\right)\right].$$

Let us suppose that a motor reaches its maximum speed in a time equal to 1.5 T; furthermore, we assume that, on the average, the rate of change of the input effect obeys the inequality

$$|A_1|\leqslant\frac{1}{2}\left|\frac{dX}{dt}\right|_{max}$$

Then, from the presented arguments we obtain

$$|y|_{max}\leqslant\frac{T}{\alpha}\frac{3}{2}\left|\frac{dX}{dt}\right|_{max}\approx 1.2. \tag{22}$$

In this way, the unit square in the (y, z) plane really is the basic operating region of the system.

We turn now to the determination of the terms of the series expansion of the approximating function $f^*(y,z)$. For this we will imagine the unit square in the (y, z) plane to be subdivided into a network of equal squares with 0.1 unit sides (Fig. 2). In this case, we will have a network with 11 × 11 nodes. Let us write down the values of $f(y_i, z_j)$ for these points (see Table 1 in the Appendix).

The first terms of the function $f^*(y, z)$, i.e., $g_0(y)$ and $h_0(z)$ we will find from the condition for a minimum in the mean-square difference for discrete values of the functions

$$D_1=\sum_{i,j}R_1^2(y_i, z_j)=\sum_{i,j}[f(y_i, z_j)-g_0(y_i)-h_0(z_j)]^2=\min. \tag{23}$$

Condition (23) is equivalent to the following two equations:

$$\frac{\partial D_1}{\partial g_0(y_i)}=-2\sum_j[f(y_i, z_j)-g_0(y_i)-h_0(z_j)]=0,$$

$$\frac{\partial D_1}{\partial h_0(z_j)}=-2\sum_i[f(y_i, z_j)-g_0(y_i)-h_0(z_j)]=0.$$

Let us impose one more condition on $g_0(y_i)$ and $h_0(z_j)$:

$$\sum_j h_0(z_j) = \sum_i g_0(y_i) = 0.$$

From here we will finally obtain $g_0(y_i)$ and $h_0(z_j)$ in the form of arithmetic means of the corresponding rows and columns in Table 1 in the Appendix:

$$g_0(y_i) = \frac{1}{n}\sum_j f(y_i, z_j), \quad h_0(z_j) = \frac{1}{n}\sum_i f(y_i, z_j). \tag{24}$$

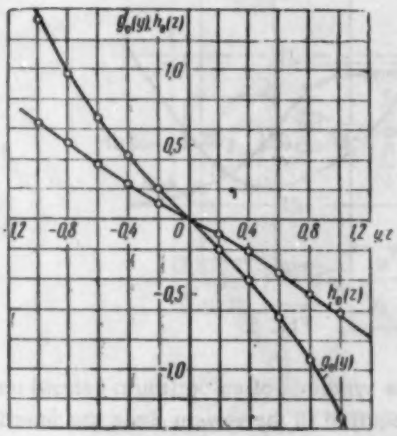

Fig. 3

The graphs of these functions are shown in Fig. 3. Let us form a table of differences (see Table 2 in the Appendix), i.e., the quantities

$$R_1(y_i, z_j) = f(y_i, z_j) - [g_0(y_i) + h_0(z_j)]. \tag{25}$$

We calculate the dispersion D_1 and the maximum relative deviation δ_1:

$$D_1 = \sum_{i,j} R_1^2(y_i, z_j) = 5{,}1,$$

$$\delta_1 = \frac{|R_1|_{\max}}{|f(y_i, z_j)|_{\max}} = \frac{0{,}65}{2{,}6} = 25\%.$$

The obtained deviation is large. Consequently, a second approximation is necessary.

We will determine the second term in the expansion of (20), i.e., the functions $g_1(y)$ and $h_1(z)$ from the following condition:

$$D_2 = \sum_{i,j} R_2^2(y_i, z_j) = \sum_{i,j} [R_1(y_i, z_j) - g_1(y_i)\, h(z_j)]^2 = \min. \tag{26}$$

Making use of the method of variations, we obtain two equations:

$$\frac{\partial D_2}{\partial g_1(y_i)} = -2\sum_j [R_1(y_i, z_j)\, h_1(z_j) - g_1(y_i)\, h_1^2(z_j)] = 0,$$

$$\frac{\partial D_2}{\partial h_1(z_j)} = -2\sum_i [R_1(y_i, z_j)\, g_1(y_i) - g_1^2(y_i)\, h_1(z_j)] = 0.$$

Solving these equations for $g_1(y_i)$ and $h_1(z_j)$, we find

$$g_1(y_i) = \frac{\sum_j R_1(y_i, z_j)\, h_1(z_j)}{\sum_j h_1^2(z_j)}, \quad h_1(z_j) = \frac{\sum_i R_1(y_i, z_j)\, g_1(y_i)}{\sum_i g_1^2(y_i)}. \tag{27}$$

The solution of these conjugate equations can be found by the method of successive approximations. First, it is necessary to pick values of $g_1(y_i)$ or $h_1(z_j)$. On the basis of these values, one can, by an iterative process, gradually approach the true functions $g_1(y_i)$ and $h_1(z_j)$. The functions determined in this way (see Appendix, Table 3) are plotted in Fig. 4. Calculations show that after the addition of the second term the dispersion $D_2 = 1.54$, and the maximum deviation $\delta_2 = 11\%$. In an absolutely analogous manner one can find the third and following terms of the expansion. The plots of $g_2(y)$ and $h_2(z)$ are shown in Fig. 5, while numerical values of these functions are entered in Table 3 in the Appendix. The addition of the third term decreases the dispersion and the maximum deviation to $D_3 = 0.059$ and $\delta_3 = 1.5\%$. Consequently, three terms of the expansion already give sufficient accuracy. The completely continuous approximating function is written in the form

$$f^*(y, z) = g_0(y) + h_0(z) + g_1(y)\,h_1(z) + g_2(y)\,h_2(z). \qquad (28)$$

Then the equations of the surface S (17) can be replaced by one equation

$$\psi(x, y, z) = x - f^*(y, z) = 0. \qquad (29)$$

At the same time the error introduced by the approximation does not exceed 1.5%. We note that Eq. (29) satisfies all the conditions of (14), (15), and (16). Hence, ψ can be used as a control function ensuring approximately optimum processes in the system.

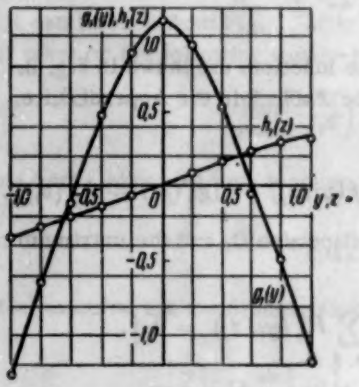

Fig. 4

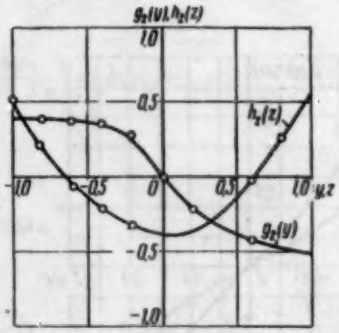

Fig. 5

On the basis of Eqs. (28) and (29) it is not difficult to carry out the synthesis of an optimum control unit. Figure 6 shows its block schematic. Only two nonlinear elements are required in the system since the functions $g_0(y)$, $h_0(z)$ and $h_1(z)$ can be linearized and reproduced by feeding the input voltages through resistances. Functions $g_1(y)$ and $h_2(z)$ are parabolic; they can be easily obtained by means of nonlinear resistances and diodes, while function $g_2(y)$ can be obtained by means of a linear element with limiting.

3. Brief Description of a Designed Optimum Control Unit

An optimum control unit intended for joint operation with an electronic model of a member or directly with the controlled member was designed on the basis of expressions (28) and (29). The system works with dc voltages. The block diagram of the control unit corresponds to the one shown in Fig. 6. The main schematic

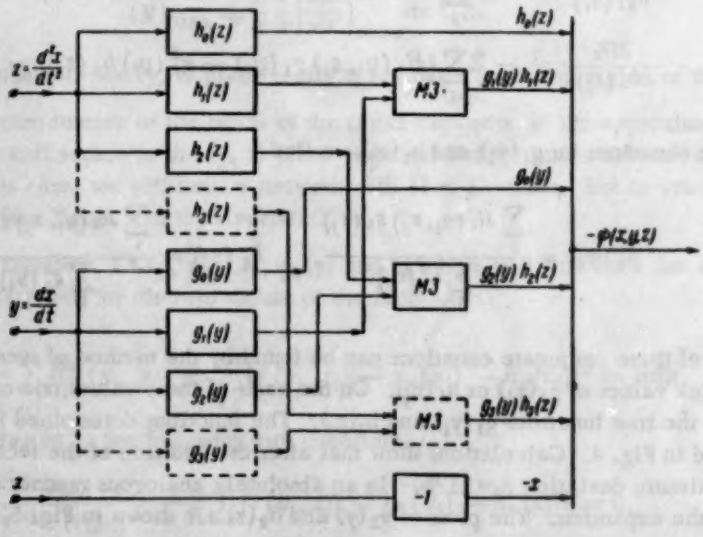

Fig. 6

of the optimum control unit is shown in Fig. 7. All the units are made up of vacuum tubes, passive components, and nonlinear resistances. The nonlinear, functional converters are made with thyrites T which, with the help of auxiliary resistances, can be easily adjusted to reproduce parabolic functions. One of the circuits of a nonlinear converter is shown in Fig. 8, a. There are also two multiplying units built with thyrites and germanium diodes. The circuit of a multiplying unit is shown in Fig. 8, b.

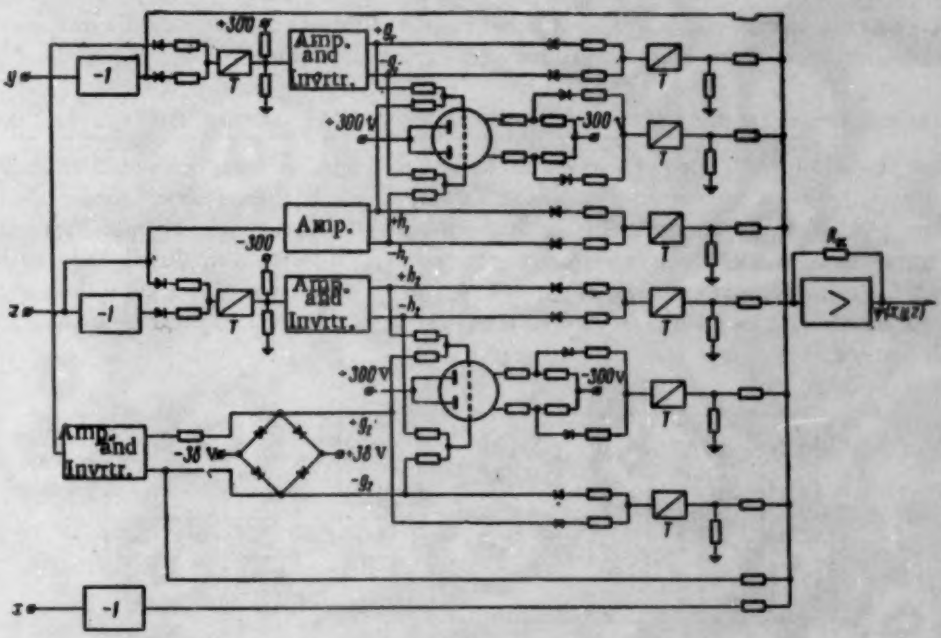

Fig. 7

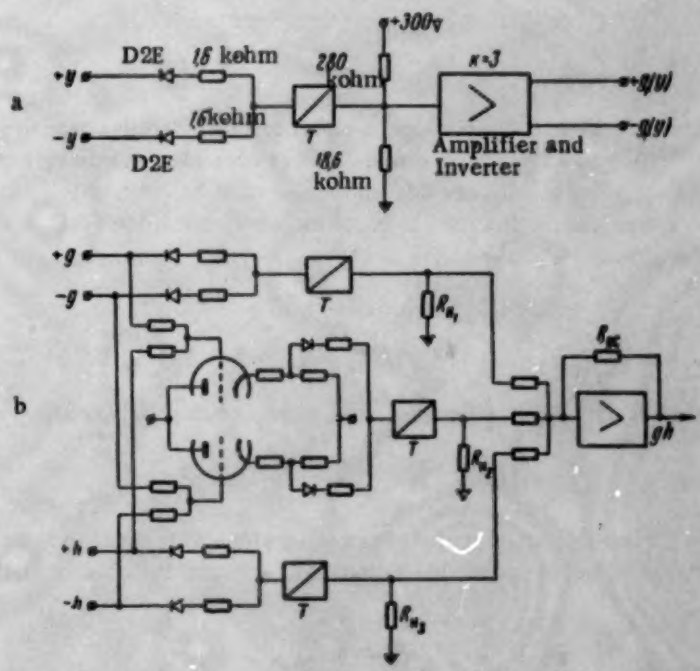

Fig. 8

To the input of the control unit are fed the quantities X_0, X, their first derivatives as well as the second derivative of the output value of X. For the multiplication of two quantities it is necessary that these quantities be positive and negative. For this purpose inverters consisting of one vacuum tube with a cathode-follower output are introduced into the circuit. In order to increase the accuracy of the solution of a problem, signals subjected to nonlinear conversion are amplified in amplifiers consisting also of one tube with heavy feedback, followed by a cathode-follower. All the signals at the output are summed in a dc amplifier. In this way the switching function ϕ (t) is obtained and is fed to the input of the relay element.

The constructed control unit is small. Another version of this circuit can be built by replacing the vacuum tubes by transistors. Such an arrangement will have miniature dimensions.

4. Results of Experiment with an Electronic Model of the Controlled Member

An experimental investigation of a model of the designed apparatus was conducted on an ÉMU-6 electronic simulator. Figure 9 shows a schematic of the model. The power unit of the system's model consists of amplifiers A-3, A-4, and A-5. The relay element consists of amplifier A-2 with a limiter and possesses a linear characteristic over the range of ± 5 v, which corresponds to the region $|\phi| < 0.05$ in phase space. In the linear region, amplifier A-2 has an amplification factor of 20, which ensures smooth operation of the system in a tracking mode at the end of a transient. The maximum voltage at the output of the relay element is ± 100 v. This voltage is taken as unit voltage.

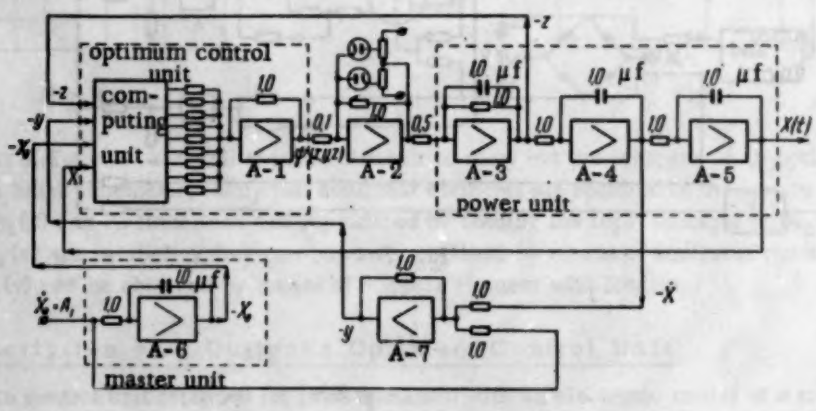

Fig. 9

An optimum system built in this way possesses a high degree of stability near the equilibrium condition. This follows from the definition of an optimum system. Let us examine the behavior of the function ϕ (x, y, z) in the vicinity of the state of equilibrium. For this we expand each function of one variable g_i and h_i into Taylor series about the origin and consider the first two terms of the expansion. After such a linearization we find the first approximation of the surface S^* at the origin of phase space:

$$[f^*(y.\,z)]_0 \approx -0.48\,y - 0.05\,z,$$

$$[\phi(x.\,y.\,z)]_0 = [x - f^*(y.\,z)]_0 \approx x + 0.48y + 0.05z.$$

The transfer function of the linearized control unit, expressed in relative units, can be written in the form

$$\phi(p) = 1 + \frac{0{,}48}{T}p + \frac{0{,}05}{T^2}p^2.$$

Let us suppose that the amplification factor of the relay element is equal to k in the linear region, in which case, taking the transfer function of the power unit into account, the characteristic equation of the closed system will be

$$p^3 + \left(k + \frac{\xi}{T^2}\right)p^2 + \frac{\tau k}{T}p + k = 0,$$

where $\xi = 0.05$ and $\tau = 0.48$.

Applying the Raus-Hurwitz criterion, we find the condition for stability of the system:

$$k > \frac{T^3}{\xi}\left(\frac{T}{\tau} - 1\right).$$

From this it can be seen that the system is always stable for a sufficiently large amplification factor $\underline{k}$. It is not difficult to show that the system is stable when $k \to \infty$. The experiment completely confirms the obtained conclusion. Moreover, the experiment showed that with increase in $\underline{k}$ the quality of the system's operation improves; the system operates smoothly in all modes and in the case when $k \to \infty$.

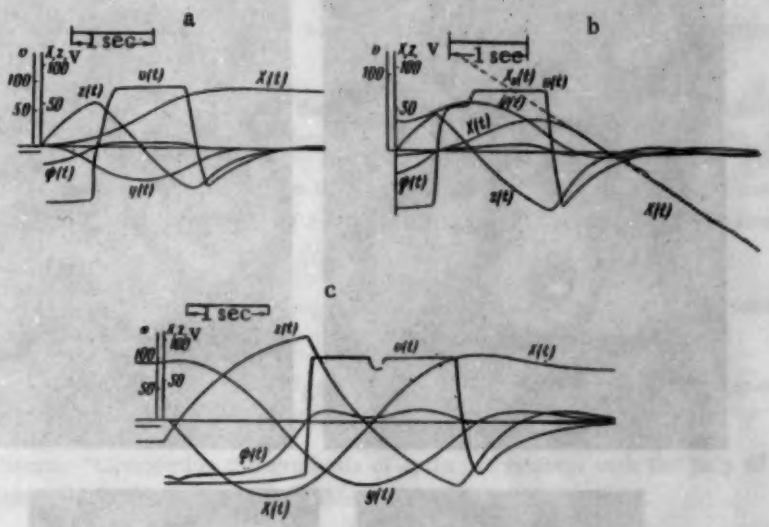

Fig. 10

The optimum system was investigated for step-type and linear inputs and for zero and nonzero initial values for the system's coordinates. The experiment showed that the designed optimum control unit ensures close to optimum transient processes for any initial conditions and for any permissible, linearly varying, input signal.

To compare the duration of transients in an optimum and a linear system, a model of a "best" linear system with the same power unit and the same variables, $\underline{x}$, $\underline{y}$, and $\underline{z}$ in the feedback loop was built. The experiment showed that for an initial unbalance of 0.75 unit the regulation time in the "best" linear system is 7.5 units (the system's time constant T is taken as a unit), while the regulation time in the optimum system for the same initial unbalance is 2.5 times less. The same relationship is preserved for the case of a linearly varying input signal. This difference can be much greater in the case of nonzero initial values of the system's coordinates.

Figure 10 shows oscillograms taken for three cases: a) correction of initial unbalance with zero initial conditions; b) correction of initial unbalance in the presence of linearly varying input signal and for zero initial conditions; c) correction of initial unbalance with nonzero initial conditions.

Shown on the oscillograms are: $X(t)$ – the output coordinate of the system, $\psi(t)$ – the control function obtained at the output of the optimum control unit, y – the error's first derivative, z – the error's second derivative, equal to the second derivative of the output quantity $\underline{x}$ taken with opposite sign, $v(t)$ – the control signal from the relay element.

From these oscillograms it can be seen that the duration of the transient in this case is somewhat larger than in a system with an ideal relay characteristic because the linear region of the relay element's characteristic does not permit the formation of an ideal third pulse and the system behaves, for small values of ψ, like a linear one. However, the deviation of the process from the optimum one is small.

Figure 11 shows photographs of the projections of the phase trajectory onto the (y, z) and (x, y) phase planes: a) the projection of the phase trajectory onto the (y, z) plane when the initial conditions are nonzero and the input signal is $X_0(t) = 0.7 + 0.25t$; b) projection of the phase trajectory on the (y, z) plane when an initial unbalance

is corrected and the initial conditions are zero; c) projection of the phase trajectory onto the (x, y) plane for the case of nonzero initial conditions and an input signal $X_0(t) = 0.7 + 0.25t$; d) projection of the phase trajectory onto the (x, y) plane when an initial unbalance is corrected and the initial conditions are zero. As can be seen from the photographs, the obtained trajectory is very close to a strictly optimum trajectory (Fig. 2).

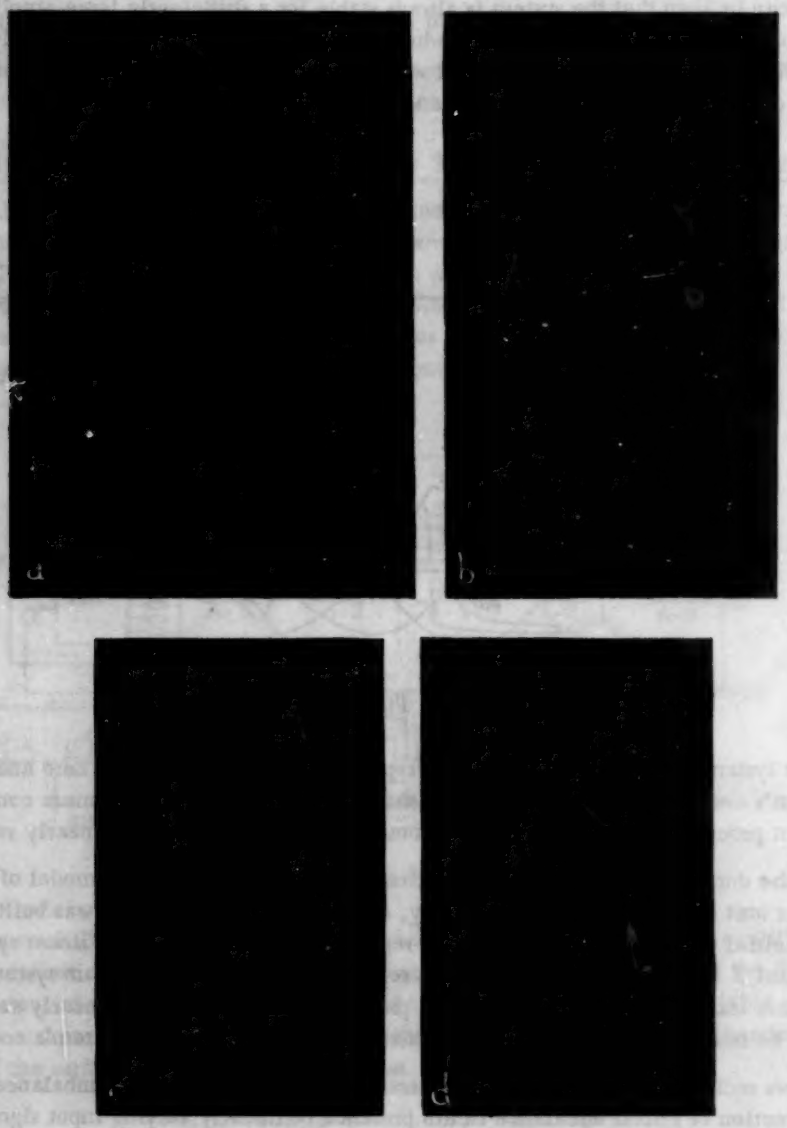

Fig. 11

SUMMARY

The described method of approximating a switching surface simplifies appreciably the problem of synthesizing an optimum system, permits the construction of an optimum control unit by relatively simple means, and ensures any required degree of accuracy without too much added complexity in the system. On the basis of this method one can construct a system which is practically identical to an ideal, optimum system. The described method can be applied to more complex systems where the hypersurface is specified by a function of a large number of variables.

The author wishes to express his gratitude to A. A. Fel'dbaum for his advice in connection with the work done on this report.

APPENDIX

See Tables 1, 2, and 3 on pp. 274 and 275.

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TABLE 1

Values of the Function $f(y_i, z)$ on the Surface S

y_i \ z_j		$j=1$	$j=2$	$j=3$	$j=4$
		−1.00	−0.80	−0.60	−0.40
$i=1$	−1.00	2.60	.22	1.90	1.70
$i=2$	−0.80	2.00	.74	1.40	1.15
$i=3$	−0.60	1.50	1.30	1.00	0.80
$i=4$	−0.40	1.00	0.80	0.65	0.52
$i=5$	−0.20	0.65	0.50	0.35	0.22
$i=6$	0.00	1.30	0.20	0.10	0.02
$i=7$	0.20	0.00	−0.03	−0.04	−0.05
$i=8$	0.40	−0.10	−0.11	−0.14	−0.17
$i=9$	0.60	−0.20	−0.23	−0.28	−0.34
$i=10$	0.80	−0.34	−0.40	−0.45	−0.56
$i=11$	1.00	−0.52	−0.60	−0.70	−0.83
$h_0(z_j)$		0.626	0.490	0.344	0.223

TABLE 2

Values $R_1(y_i, z_j) = f(y_i, z_j) - g_0(y_i) - h_0(z_j)$

$g_0(y_i)$ \ $h_0(z_j)$		$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$
		0.626	0.490	0.344	0.223	0.109	0.000	−0.109	−0.223	−0.344	−0.490	−0.626
$i=1$	1.320	0.65	0.41	0.24	0.16	−0.06	−0.20	−0.26	−0.27	−0.28	−0.23	−0.17
$i=2$	0.958	0.42	0.29	0.10	−0.03	−0.07	−0.16	−0.17	−0.17	−0.16	−0.07	0.01
$i=3$	0.663	0.21	0.15	0.01	−0.09	−0.12	−0.11	−0.10	−0.10	−0.04	0.06	0.16
$i=4$	0.395	−0.02	−0.09	−0.09	−0.10	−0.12	−0.12	−0.09	0.00	0.09	0.21	0.33
$i=5$	0.196	−0.17	−0.18	−0.19	−0.20	−0.16	−0.10	−0.02	0.07	0.19	0.32	0.43
$i=6$	0.000	−0.33	−0.29	−0.24	−0.20	−0.11	−0.00	0.11	0.20	0.24	0.29	0.33
$i=7$	−0.196	−0.43	−0.32	−0.19	−0.07	0.02	0.10	0.16	0.20	0.19	0.18	0.17
$i=8$	−0.395	−0.33	−0.21	−0.09	−0.00	0.09	0.12	0.12	0.10	0.09	0.09	0.02
$i=9$	−0.663	−0.16	−0.06	0.04	0.10	0.10	0.11	0.12	0.09	0.01	−0.15	−0.21
$i=10$	−0.956	−0.01	0.07	0.16	0.17	0.17	0.16	0.07	0.03	−0.10	−0.29	−0.42
$i=11$	−1.320	0.17	0.23	0.28	0.27	0.26	0.20	0.06	−0.16	−0 24	−0.41	−0.65

TABLE 3

Values of Functions g_1, h_1, g_2 and h_2

i, j	1	2	3	4	5	6	7	8	9	10	11
$g_1(y_i)$	−1.27	−0.65	−0.13	0.46	0.88	1.08	0.93	0.52	−0.06	−0.49	−1.18
$h_1(z_j)$	−0.35	−0.28	−0.21	−0.15	−0.06	0.01	0.09	0.17	0.21	0.26	0.31
$g_2(y_i)$	0.39	0.39	0.37	0.35	0.28	0.04	−0.22	−0.27	−0.34	−0.42	−0.48
$h_2(z_j)$	0.50	0.20	−0.06	−0.22	−0.30	−0.39	−0.32	−0.18	−0.02	−0.26	0.57

$j=5$	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$g_0 f_i$
−0,20	0;00	0,20	0,40	0,60	0,80	1,00	
1.37	1.12	0,95	0.83	0.70	0.60	0.52	1.320
1.00	0,80	0.68	0.56	0.45	0.40	0.34	0.956
0.65	0.55	0.45	0.34	0.28	0.23	0.20	0.663
0.38	0.28	0.20	0.17	0.14	0.11	0.10	0.395
0.15	0.10	0.07	0.05	0.04	0.03	0.00	0.196
0.00	0.00	0.00	−0.02	−0.10	−0.20	−0.30	0.000
−0.07	−0.10	−0.15	−0.22	−0.35	−0.50	−0.65	−0.196
−0.20	−0.28	−0.38	−0.52	−0.65	−0.80	−1.00	−0.395
−0.45	−0.55	−0.65	−0.80	−1.00	−1.30	−1.50	−0.663
−0.68	−0.80	−1.00	−1.15	−1.40	−1.74	−2.00	−0.956
−0.95	−1.12	−1.37	−1.70	−1.90	−2.22	−2.60	−1.320
0.109	0.00	−0.109	−0.223	−0.344	−0.490	−0.626	

CONCERNING THE REALIZATION OF OPTIMUM WEIGHTING FUNCTIONS OF PULSE SERVOSYSTEMS

V. P. Perov

(Leningrad)

Simple techniques for the approximate realization of optimum weighting functions of pulse servosystems are given for several particular cases.

Statement of Problem

We will assume that acting on the input of a pulse servosystem is a regular signal, expressed as a polynomial of degree r, and random noise having a correlation function

$$R_{pp}[nT] = \begin{cases} c^2 \text{ when } n = 0, \\ 0 \text{ when } n \neq 0, \end{cases} \tag{1}$$

where T is the pulse repetition period in the pulse system, and n is a whole number.

Under these conditions the pulse servosystem's weighting function, optimum from the point of view of minimum mean square and given dynamic errors, for a limited transient time, can be represented by the expressions [1]:

a) when r = 1

$$K_c[n, \varepsilon] = \frac{1}{(N+1)(N+2)}\left\{[1 + c_0(\varepsilon)]\frac{2N+1}{3} - n + [\varepsilon + c_1(\varepsilon)]\left(1 - \frac{2n}{N}\right)\right\} \quad (0 \leqslant n \leqslant N),$$
$$K_c[n, \varepsilon] = 0 \quad (0 > n > N); \tag{2}$$

b) when r = 2

$$K_c[n, \varepsilon] = \frac{3}{(N+1)(N+2)(N+3)}\left\{[1 + c_0(\varepsilon)][(3N^2 + 3N + 2) - \right.$$
$$- 6(2N+1)n + 10n^2] + [\varepsilon + c_1(\varepsilon)]\left[6(2N+1) - \frac{4(2N+1)(8N-3)}{N(N-1)}n + \right.$$
$$\left.\left. + \frac{60}{N-1}n^2\right] + [\varepsilon^2 + 2c_2(\varepsilon)]\left[10 - \frac{60}{N-1}n + \frac{60}{N(N-1)}n^2\right]\right\} \quad (0 \leqslant n \leqslant N),$$
$$K_c[n, \varepsilon] = 0 \quad (0 > n > N). \tag{3}$$

Here $n + \varepsilon = t$ is time expressed in pulse repetition periods, where n is a whole number and ε is a time parameter varying over the range 0–1, N is the specified transient time of the system, $K_c[n, \varepsilon]$ is the weighting function of the pulse system which is assumed to be closed, $c_0(\varepsilon)$, $c_1(\varepsilon)$, and $c_2(\varepsilon)$ are the given coefficients of the dynamic error $d[n, \varepsilon]$. The dynamic error is expressed in terms of the coefficient $c_1(\varepsilon)$ in the following way:

$$d[n, \varepsilon] = c_0(\varepsilon) f[n] + c_1(\varepsilon) \dot{f}[n] + c_2(\varepsilon) \ddot{f}[n]. \tag{4}$$

In (4) $f[n]$, $\dot{f}[n]$, and $\tilde{f}[n]$ denote the regular signal and its derivatives.

Below are presented the simplest techniques for the approximate realization of the weighting functions in (2) for the cases when

$$c_0(\varepsilon) = c_1(\varepsilon) = 0, \tag{5}$$

$$c_0(\varepsilon) = 0, \quad c_1(\varepsilon) \neq 0, \tag{6}$$

and the weighting functions in (3) for the cases when

$$c_0(\varepsilon) = c_1(\varepsilon) = c_2(\varepsilon) = 0, \tag{7}$$

$$c_0(\varepsilon) = c_1(\varepsilon) = 0, \quad c_2(\varepsilon) \neq 0. \tag{8}$$

These methods are based on the analytical approximation of optimum weighting functions by desirable functions with the subsequent conversion to transfer functions [1].

1. Realization of Optimum Weighting Functions Obtained by Means of Equation (2) for the Case $c_0(\epsilon) = c_1(\epsilon) = 0$

It was shown in [1] that weighting function (2) for the case when $c_0(\epsilon) = c_1(\epsilon) = 0$, can be approximately realized by means of a system such as the one shown in block diagram form in Fig. 1, while the values of parameters K_0 and K_1 are determined from the equations

$$K_1 = \lim_{q \to 0} (e^q - 1)^2 K_p^*(q, 0), \tag{9}$$

$$K_0 = \lim_{q \to 0} \left[(1 - e^{-q}) K_p^*(q, 0) - \frac{K_1}{e^q - 1} \right], \tag{10}$$

where

$$K_p^*(q, 0) = \frac{\sum_{n=0}^{N} K_C[n, 0] e^{-qn}}{1 - e^{-q} \sum_{n=0}^{N} K_C[n, 1] e^{-qn}} \tag{11}$$

Equations (9) and (10) are inconvenient to use in practice because for each particular case it is necessary to repeat the operation of determining limits. To avoid such an inconvenience we determine the general form of the limits expressed by equalities (9) and (10).

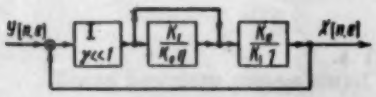

Fig. 1. Block schematic of system for the approximate realization of weighting functions (2) when $c_0(\epsilon) = c_1(\epsilon) = 0$. γ = duty ratio of pulse element $Y[n, \epsilon]$, $X[n, \epsilon]$ are the input and output functions of the pulse system.

Substituting $K_C[n, \epsilon]$ from (2), for the case when $c_0(\epsilon) = c_1(\epsilon) = 0$, into (11) and the resulting expression for $K_p^*(q, 0)$ into (9) and (10), we find

$$K_1 = \frac{12}{(N+2)(N+3)}, \qquad K_0 = \frac{12}{5} \frac{(2N+3)}{(N+2)(N+3)}. \tag{12}$$

Equations (12) permit one to determine directly the required values of parameters K_0 and K_1 of the desired system.

We note that the desired system obtained in this way ensures that the first two coefficients of the dynamic error $[c_{0d}(\epsilon) = c_{1d}(\epsilon) = 0]$ are equal to zero. At the same time the mean-square error is equal to

$$\sigma_d = \sqrt{c^2 \frac{2K_1 + K_0(2K_0 - 3K_1) + 2K_1(2K_0 - K_1)\varepsilon + 2K_1^2\varepsilon^2}{K_0(4 - K_1 - 2K_0)}}. \tag{13}$$

Expression (13) can be obtained by means of the formula for the mean-square error given in [2], if the concrete form of the desired pulse system's transfer function is taken into account.

2. Realization of Optimum Weighting Functions Obtained by Means of Equation (3) for the Case $c_0(\epsilon) = c_1(\epsilon) = c_2(\epsilon) = 0$

Weighting function (3), for the case when $c_0(\epsilon) = c_1(\epsilon) = c_2(\epsilon) = 0$, can be approximately realized by means of the system whose block diagram is shown in Fig. 2. The values of parameters K_0, K_1, and K_2 in this case are determined from the equalities [1]

$$K_2 = \lim_{q \to 0} \frac{1}{2} (e^q - 1)^3 K_p^*(q, 0), \tag{14}$$

$$K_1 = \lim_{q \to 0} (e^q - 1)\left[(1 - e^{-q}) K_p^*(q, 0) - K_2 \frac{(e^q + 1)}{(e^q - 1)^2}\right], \tag{15}$$

$$K_0 = \lim_{q \to 0}\left[(1 - e^{-q}) K_p^*(q, 0) - \frac{K_1}{e^q - 1} - \frac{K_2 (e^q + 1)}{(e^q - 1)^2}\right]. \tag{16}$$

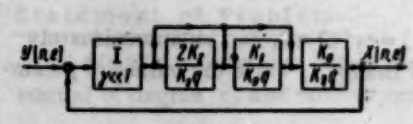

Fig. 2. Block schematic of circuit for the approximate realization of weighting functions (3) when $c_0(\epsilon) = c_1(\epsilon) = c_2(\epsilon) = 0$.

The limits expressed by Eqs. (14), (15), and (16) can be found in a general form. In fact, substituting $K_C[n, 0]$ from (3), when $c_0(\epsilon) = c_1(\epsilon) = c_2(\epsilon) = 0$ into (11) and the resulting expression for $K_p^*(q, 0)$ into (14), (15), and (16), we can find

$$K_2 = \frac{60}{(N+2)(N+3)(N+4)},$$

$$K_1 = \frac{180}{7} \frac{2N+3}{(N+2)(N+3)(N+4)},$$

$$K_0 = \frac{30}{49} \frac{15N^2 + 45N + 46}{(N+2)(N+3)(N+4)}. \tag{17}$$

From (17) it is simple to determine the required values of the parameters of the desired system.

We note that in the given case the desired system guarantees that the first three coefficients of the dynamic error are equal to zero:

$$c_{0\,d}(\epsilon) = c_{1\,d}(\epsilon) = c_{2\,d}(\epsilon) = 0. \tag{18}$$

The mean square of the random error of the desired system can be calculated from the formula

$$\sigma^2 = \frac{\sigma^2}{\Delta}\Big\{K_1(3K_0K_1 - 2K_0^2 - 2K_1) + K_2K_0(4 - 2K_0 - K_1) + \\ + 2K_1(2K_0 - K_1)(K_2 - K_1)\epsilon + 2K_1[K_2(3K_1 - 2K_0 - K_2) - K_1^2]\epsilon^2 + \\ + 4K_1K_2(K_2 - K_1)\epsilon^3 - 2K_1K_2^2\epsilon^4\Big\}, \tag{19}$$

where

$$\Delta = (2K_0 + K_1 - 4)(K_0K_1 + K_0K_2 - 2K_2). \tag{20}$$

3. Realization of Optimum Weighting Functions Obtained with the Help of Equation (2) in the Case when $c_0(\epsilon) = 0$, $c_1(\epsilon) \neq 0$.

We will examine the problem of realizing optimum weighting functions of (2) when $c_0(\epsilon) = 0$ and $c_1(\epsilon) \neq 0$, assuming that

$$c_1(\varepsilon) = b_1(N + 2\varepsilon), \tag{21}$$

where b_1 is an arbitrary constant coefficient.

TABLE 1

n	$K_c[n, \varepsilon]$			
	$N=1$ $b_1=-0.100$	$N=1$ $b_1=-0.167$	$N=1$ $b_1=-0.250$	$N=2$ $b_1=-0.055$
0	$0.90+0.80\varepsilon$	$0.83+0.67\varepsilon$	$0.75+0.50\varepsilon$	$0.78+0.44\varepsilon$
1	$0.10-0.80\varepsilon$	$0.17-0.67\varepsilon$	$0.25-0.50\varepsilon$	0.33
2	0	0	0	$-0.11-0.44\varepsilon$

Weighting functions obtained from (2) for different values of N and b_1 are listed in Table 1.

The corresponding weighting functions of open-loop systems, obtained with the help of the relationship [1]

$$K_p[n, \varepsilon] = K_c[n, \varepsilon] + \sum_{l=0}^{n-1} K_p[l, 1]\, K_c[n-l-1, \varepsilon], \tag{22}$$

are listed in Table 2 and are plotted in Fig. 3.

TABLE 2

n	$K_p[n, \varepsilon]$			
	$N=1$ $b_1=-0.100$	$N=1$ $b_1=-0.167$	$N=1$ $b_1=-0.250$	$N=1$ $b_1=-0.055$
0	$0.90+0.80\varepsilon$	$0.83+0.67\varepsilon$	$0.75+0.50\varepsilon$	$0.78+0.44\varepsilon$
1	$1.63+0.80\varepsilon$	$1.41+0.33\varepsilon$	$1.19+0.12\varepsilon$	$1.28+0.54\varepsilon$
2	$2.14+0.39\varepsilon$	$1.70+0.16\varepsilon$	$1.30+0.03\varepsilon$	$1.72+0.38\varepsilon$
3	$2.50+0.27\varepsilon$	$1.85+0.08\varepsilon$	$1.32+0.01\varepsilon$	$2.09+0.39\varepsilon$
4	$2.75+0.19\varepsilon$	$1.93+0.04\varepsilon$	1.33	$2.42+0.30\varepsilon$
5	$2.93+0.13\varepsilon$	$1.98+0.02\varepsilon$	1.33	$2.74+0.30\varepsilon$
6	$3.06+0.09\varepsilon$	$2.01+0.01\varepsilon$	1.33	$3.00+0.20\varepsilon$

These functions possess small discontinuities but can be approximated by exponentials. This permits one to select the desired weighting function of the open-loop system in the following form:

$$K_{pd}[n, \varepsilon] = K_0 + c' e^{-\alpha(n+\varepsilon)}, \tag{23}$$

where K_0, c', and α are constant parameters.

The transfer function K(q) of the equivalent continuous part (ECP) of the system is expressed in terms of $K_p[n, \epsilon]$ in the following manner [1]:

$$K(q) = \int_0^1 \sum_{n=0}^{\infty} K_p[n, \varepsilon]\, e^{-qn} e^{-q\varepsilon}\, d\varepsilon. \tag{24}$$

In the given case, we obtain, on the basis of (24) and (23), the following expression for the transfer function of the ECP:

$$K_d(q) = \frac{(K_0 + c')q + \alpha K_0}{q(q+\alpha)}. \tag{25}$$

The block schematics of pulse systems corresponding to this expression are shown in Fig. 4.

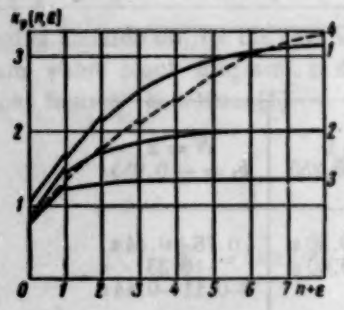

Fig. 3. Weighting functions of open-loop pulse systems, corresponding to the weighting functions of the closed-loop systems obtained from (2). 1) $c_1(\epsilon) = -0.100(1 + 2\epsilon)$ when $N = 1$; 2) $c_1(\epsilon) = -0.167(1 + 2\epsilon)$ when $N = 1$; 3) $c_1(\epsilon) = -0.250(1 + 2\epsilon)$ when $N = 1$; 4) $c_1(\epsilon) = -0.055(2 + 2\epsilon)$ when $N = 2$.

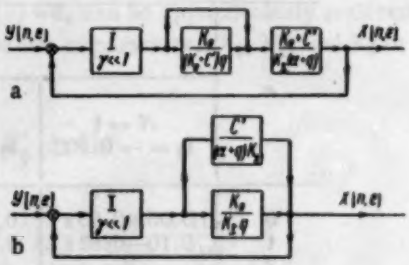

Fig. 4. Block schematics of systems for the approximate realization of the weighting functions of (2) when $c_0(\epsilon) = 0$, $c_1(\epsilon) = b_1(N + 2\epsilon)$.

It remains to find expressions for the parameters K_0, c', and α of the desired system. In determining these, one can start with the conditions for the initial and limiting coincidence of the desired and optimum weighting functions of the open-loop systems, as well as the condition for the equality of the areas bounded by these weighting functions. The indicated conditions are written in the following manner:

$$\lim_{n\to\infty} K_{pd}[n, 0] = \lim_{n\to\infty} K_p[n, 0], \tag{26}$$

$$K_{pd}[0, 0] = K_p[0, 0], \tag{27}$$

$$\int_0^1 \sum_{n=0}^{\infty} K_{pd}[n, \varepsilon]\, d\varepsilon = \int_0^1 \sum_{n=0}^{\infty} K_p[n, \varepsilon]\, d\varepsilon, \tag{28}$$

where $K_{pd}[n, \epsilon]$ and $K_p[n, \epsilon]$ are the weighting functions of the desired and optimum open-loop systems.

On the basis of the theorem concerning the limiting value of a singularity, and the area bounded by a singularity, and taking Eqs. (23), (22), and (2) into account, we obtain

$$\lim K_{pd}[n, 0] = K_0, \tag{29}$$

$$K_{pd}[0, 0] = K_0 + c', \tag{30}$$

$$\int_0^1 \sum_{n=0}^{\infty} K_{pd}[n, \varepsilon]\, d\varepsilon = \frac{c'}{\alpha} + \lim_{q\to 0} \frac{K_0 e^q}{e^q - 1}, \tag{31}$$

$$\lim_{n\to\infty} K_p[n, 0] = \lim_{q\to 0} (e^q - 1) K_p^*(q, 0), \tag{32}$$

$$K_p[0, 0] = K_s[0, 0] = \frac{2[2N + 1 + 3c_1(\varepsilon)]}{(N+1)(N+2)}, \tag{33}$$

$$\int_0^1 \sum_{n=0}^{\infty} K_p[n, \varepsilon]\, d\varepsilon = \lim_{q\to 0} \int_0^1 K_p^*(q, \varepsilon)\, d\varepsilon. \tag{34}$$

From (32), (29), and (26), it follows that

$$K_0 = \lim_{q \to 0} (e^q - 1)\, K_p^*(q, 0). \tag{35}$$

From (33), (30), (27), and (21) it follows that

$$c' = \frac{2\,[2N + 1 + 3b_1 N]}{(N+1)(N+2)} - K_0. \tag{36}$$

From (34), (31), and (28) it follows that

$$\alpha = \frac{c'}{s}, \tag{37}$$

where

$$s = \lim_{q \to 0} \left[\frac{-K_0 e^q}{e^q - 1} + \int_0^1 K_p^*(q, \varepsilon)\, d\varepsilon \right]. \tag{38}$$

The limits expressed by Eqs. (35) and (36) can be determined in a general form by substituting into these equations the general expression for $K_p^*(q, \epsilon)$, which results from (2) and (11).

As a result, instead of (35) and (39), we obtain

$$K_0 = \frac{-N - 2\varepsilon}{c_1(\varepsilon)(N+2)}, \tag{39}$$

$$s = -\frac{N+1}{N+2} - \frac{N+2\varepsilon}{2c_1(\varepsilon)} - \frac{(N+3)(N+2\varepsilon)^2}{12\, c_1^2(\varepsilon)(N+2)} \tag{40}$$

and in accordance with (21)

$$K_0 = -\frac{1}{b_1(N+2)}, \tag{39a}$$

$$s = -\frac{N+1}{N+2} - \frac{1}{2b_1} - \frac{N+3}{12 b_1^2 (N+2)}. \tag{40a}$$

Thus, in the case being considered the circuit of the desired system can be chosen in accordance with one of the circuits shown in Fig. 4, while the required values of its parameters can be easily found from Eqs. (39a), (36), (37), and (40a).

It can be shown that for the desired system chosen in this way the coefficients of the dynamic error will have the form:

$$c_{0\,d}(\varepsilon) = 0, \quad c_{1\,d}(\varepsilon) = \frac{K_0 - 1}{K_0} - \frac{c'}{K_0(e^\alpha - 1)} + \frac{c' e^{-\alpha\varepsilon}}{K_0(1 - e^{-\alpha})} - \varepsilon. \tag{41}$$

At the same time, the mean-square error is determined by the expression

$$\sigma_d = \sqrt{c^2 \frac{K_0[(1-e^\alpha)(1+e^\alpha - c') - K_0(1+e^\alpha)] - 2K_0 c' e^\alpha (1+e^\alpha) e^{-\alpha\varepsilon} - 2c'^2 e^{2\alpha} e^{-2\alpha\varepsilon}}{(K_0 + c' + e^\alpha - 1)[K_0(1+e^\alpha) - 2(1 - c' + e^\alpha)]}}. \tag{42}$$

4. Realization of Optimum Weighting Functions Obtained with the Help of Eq. (3) for the Case $c_0(\epsilon) = c_1(\epsilon) = 0$, $c_2(\epsilon) \neq 0$

Let us examine the problem of realizing the weighting functions of (3) for the case when $c_0(\epsilon) = c_1(\epsilon) = 0$, and $c_2(\epsilon) \neq 0$, assuming that

$$c_2(\varepsilon) = b_2[N(N-1) + 6N\varepsilon + 6\varepsilon^2], \tag{43}$$

where b_2 is an arbitrary constant coefficient.

In the case in question the weighting functions of open-loop systems obtained on the basis of (3) and (22) for various values of b_2 and N_1, tend exponentially to inclined straight lines, coinciding with them in the limit. This serves as a basis for the choice of the desired weighting function of the open-loop system in the form

$$K_{pd}[n, \varepsilon] = K_0 + K_1(n + \varepsilon) + c'e^{-\alpha(n+\varepsilon)}. \tag{44}$$

For the transfer function of the ECP of the desired system we obtain, on the basis of (44) and (24), the expression

$$K_d(q) = \frac{(K_0 + c')q^2 + (K_0\alpha + K_1)q + K_1\alpha}{q^2(q+\alpha)}.$$

The block schematics of pulse systems corresponding to this are shown in Fig. 5.

It remains to find expressions for the parameters K_0, K_1, c', and α. In determining them one can start from the conditions for the coincidence of the limiting and initial values of the desired and optimum weighting functions of open-loop systems, from the equality of the areas bounded by the curves of these functions, and from the condition for the coincidence of the limiting values of the first deviations from these weighting functions. The indicated conditions are written in the following manner:

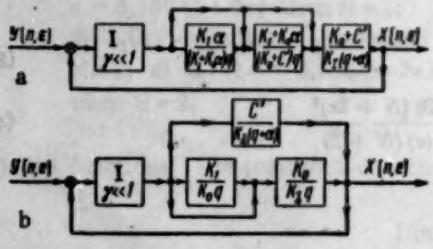

Fig. 5. Block schematics of systems approximately realizing the weighting functions of (3) when $c_0(\epsilon) = c_1(\epsilon) = 0$, $c_2(\epsilon) = b_2[N \times (N-1) + 6N\epsilon + 6\epsilon^2]$.

$$\begin{cases} \lim\limits_{n\to\infty} K_{pd}[n, 0] = \lim\limits_{n\to\infty} K_p[n, 0], \\ \lim\limits_{n\to\infty} \Delta K_{p\,d}[n, 0] = \lim\limits_{n\to\infty} \Delta K_p[n, 0], \\ K_{p\,d}[0, 0] = K_p[0, 0] = K_c[0, 0], \\ \int\limits_0^1 \sum\limits_{n=0}^{\infty} K_{p\,d}[n, \varepsilon]\, d\varepsilon = \int\limits_0^1 \sum\limits_{n=0}^{\infty} K_p[n, \varepsilon]\, d\varepsilon. \end{cases} \tag{45}$$

From (45), on the basis of the theorem concerning the limiting value of a singularity and concerning the area bounded by a singularity, and taking into account equations (43), (22), and (3), we obtain

$$K_1 = \lim_{q\to 0}(e^q - 1)K_p^*(q, 0), \tag{46}$$

$$K_0 = \lim_{q\to 0}\left[(1 - e^{-q})K_p^*(q, 0) - \frac{K_1}{e^q - 1}\right]. \tag{47}$$

$$c' = K_c[0, 0] - K_0 = \frac{3(3N^2 + 3N + 2) + 20c_2(N-1)N}{(N+1)(N+2)(N+3)} - K_0, \tag{48}$$

$$\alpha = \frac{c'}{s}, \tag{49}$$

where

$$s = \lim_{q\to 0}\left[\int_0^1 K_p^*(q, \varepsilon)\, d\varepsilon - \left(K_0 - \frac{K_1}{2}\right)\frac{e^q}{e^q - 1} - \frac{K_1 e^q}{(e^q - 1)^2}\right]. \tag{50}$$

The limits expressed by Eqs. (46), (47), and (50) can be determined in a general form by substituting into these equations the expression for $K_p^*(q, \epsilon)$ obtained from (3) and (11) when $c_0(\epsilon) = c_1(\epsilon) = 0$, and $c_2(\epsilon) \neq 0$. As a result, instead of (46), (47), and (50), we find

$$K_0 = \frac{6\,(2\,(2N+3) - N\,(N-1)\,[3N+2+(N-1)\,N\,(N+4)])}{5\,(N+2)\,(N+3)} -$$
$$- \frac{(N-1)N(3N+2) + 2\,(N-1)^2\,N^2\,(N+4)}{10b_2\,(N+2)\,(N+3)} - \frac{(N-1)^2\,N^2\,(N+4)}{120\,b_2^2\,(N+2)\,(N+3)}, \qquad (51)$$

$$K_1 = \frac{12\,(1+N-N^2)}{(N+2)\,(N+3)} - \frac{(N-1)\,N}{b_2\,(N+2)\,(N+3)},$$

$$s = \left\{\frac{(N-1)\,(-6N^2+6N+3)}{(N+2)\,(N+3)} - \frac{(N-1)\,N\,(6N^2-1)}{12\,b_2\,(N+2)\,(N+3)}\right\} -$$
$$- K_0\left\{\frac{(N-1)\,(6-4N-N^2)}{10} - \frac{(N-1)\,N\,(N+4)}{120\,b_2}\right\} -$$
$$- K_1\left\{\frac{(N-1)\,[14\,(2N-1) - N\,(N+4)\,(8N-9)]}{140} - \frac{(N-1)\,N\,(N+4)\,(8N-9)}{1800\,b_2}\right\},$$

where b_2 is the coefficient of $c_2(\varepsilon)$ in expression (43).

Thus, in the case being considered the circuit of the desired system can be selected from one of the circuits shown in Fig. 5, while the required values of its parameters are found by means of Eqs. (51), (49), and (48).

It can be shown that the desired system selected in this way will ensure the following coefficients of the dynamic error:

$$c_{0\,d}(\varepsilon) = c_{1\,d}(\varepsilon) = 0,\ c_{2d}(\varepsilon) = \frac{K_0 - 1}{K_1} + \frac{c'}{K_1}\,\frac{e^{\alpha(1-\varepsilon)} - 1}{e^{\alpha} - 1} - \frac{K_1 - 2K_0}{2K_1}\,\varepsilon - 0{,}5\,\varepsilon^2.$$

The mean-square error of the desired system can be calculated from the following formula:

$$\sigma = \sqrt{c^2 \sum_{n=0}^{N} K_c^2[n, \varepsilon]}, \qquad (52)$$

after substituting into it the function $K_c[n, \epsilon] = K_{cd}[n, \epsilon]$ determined by means of the recurrent relationship

$$K_{cd}[n, \varepsilon] = K_{p\,d}[n, \varepsilon] - \sum_{l=0}^{n-1} K_{p\,d}[l, 1]\,K_{cd}[n-l-1, \varepsilon],$$

where

$$K_{p\,d}[n, \varepsilon] = K_0 + K_1(n+\varepsilon) + c'e^{-\alpha(n+\varepsilon)}.$$

Example. Let us suppose that acting on the input of the pulse servosystem of a circular scan radar station is a random noise signal with correlation function (1) and a regular signal which, in this case, is the coordinate of the target.

TABLE 3

ε	For desired system		For optimum system	
	$c_{1d}(\epsilon)$	$\frac{1}{c}\sigma_d(\epsilon)$	$c_1(\epsilon)$	$\frac{1}{c}\sigma(\epsilon)$
0	—0.10	0.81	—0.20	0.81
0.25	—0.15	0.91	—0.25	0.91
0.50	—0.23	1.01	—0.30	1.01
0.75	—0.31	1.12	—0.35	1.12
1.00	—0.40	1.22	—0.40	1.22

It is required to choose a system such that it will ensure a minimum mean-square, tracking error when the time of the transient process is equal to two repetition periods (N = 2), and when the coefficients of the dynamic error are specified in the following manner.

$$c_0(\varepsilon) = 0,\ c_1(\varepsilon) = b_1(N + 2\varepsilon). \qquad (53)$$
$$(b_1 = -0.1).$$

To solve this problem we make use of the results obtained in Sect.3, whence it follows that the desired system can be selected from one of the circuits shown in Fig. 4.

Equations (39a), (36), (37), and (40a) are used to find the values of the parameters K_0, c', and α:

$$K_0 = 2.50,\ c' = -1.76,\ \alpha = 0.29. \tag{54}$$

On the basis of (54), (41), and (42), we obtain for the desired system

$$c_{0\,d}(\varepsilon) = 0,\ c_{1\,d}(\varepsilon) = 2.70 - 2.80\,e^{-0.29\varepsilon} - \varepsilon,$$
$$\sigma_d = c\sqrt{7.17 - 10.92\,e^{-0.29\varepsilon} + 4.41\,e^{-0.58\,\varepsilon}}. \tag{55}$$

In the analogous case, i.e., when $N = 2$, $c_0(\varepsilon) = 0$, $c_1(\varepsilon) = -0.1\,(N + 2\varepsilon)$, we obtain for the optimum system, taking (2) and (52) into account,

$$c_0(\varepsilon) = \sum_{n=0}^{N} K_C[n, \varepsilon] - 1 = 0,\ c_1(\varepsilon) = -\sum_{n=0}^{N} nK_C[n, \varepsilon] - \varepsilon = -0.1\,(2 + 2\varepsilon), \tag{56}$$
$$\sigma = c\sqrt{0.654 + 0.623\,\varepsilon + 0.231\,\varepsilon^2}.$$

Values of σ_d and σ, as well as $c_{1d}(\epsilon)$ and $c_1(\epsilon)$, calculated from (55) and (56) for different ϵ, are presented in Table 3, whence it follows that the dynamic tracking errors of the optimum and desired pulse systems differ by an insignificant amount, while their mean-square errors coincide within limits of the accuracy of calculations.

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* See English translation.

MATHEMATICAL PROBLEMS ASSOCIATED WITH THE AUTOMATIC CONTROL OF A MILLING MACHINE*

I. I. Éterman

(Penza)

Mathematical problems arising in the automatic control of a milling machine by means of a digital computer are considered. The machine's structure and parameters are analyzed. Formulas are given for the processing of data for conversion into control pulses.

The direct control of milling machines with the help of output pulses produced by a high-speed digital computer is not expedient. It is proposed to limit the function of the digital computer to the processing of input data concerning the contour to be worked into output data of minimum content convenient for subsequent conversion into a series of control signals. This information can be recorded on magnetic tape, perforated tape, punched cards, etc., after being checked and, if necessary, corrected and can be stored until required. The above-mentioned digital device is, on the basis of its functions, a specialized form of digital computer (SDC) with a large capacity, intended for serving a large number of factories.

The direct control of machines must be carried out with the help of a small and quite simple digital unit—an auxiliary unit, henceforth referred to as an AU. The functions of an AU consists of converting the output data from the SDC, which we will consider, for the sake of being definite, to be recorded on a perforated tape, into a series of pulses for controlling the machine.

It will be subsequently shown that the mathematical functions of the AU can be reduced to the adding and comparing of codes as well as carrying out several service operations. The proposed distribution of functions between the SDC and the AU must ensure a solution to the problem of controlling machines, which is profitable both from the technical and economic point of view.

The problem of working products in two dimensions is considered in this paper.

1. Basic Relationships

To produce an object having a certain prescribed shape it is necessary that the center of the milling cutter travel along an equidistant, i.e., along a curve representing the locus of points situated at a distance d ($d > 0$) (along the perpendicular) from the given smooth contour Γ (Fig. 1).

The equidistant curve will be external (Γ^*) or internal (Γ^{**}) depending on whether the external or internal perpendicular distance is chosen.

Hence, we can write

$$L_d\Gamma = \Gamma^*, \tag{1}$$

* Part of the results contained in this paper were reported by the author on November 14, 1957, at the All-Union Conference on the Programmed Control of Metal-Cutting Machines.

$$L_{-d}\Gamma = \Gamma^{**}. \tag{2}$$

Expression (1) shows that the arc Γ is transformed into the arc Γ^* by means of the external transformation with parameter $\underline{d}$. Analogously, expression (2) shows that the internal transformation with parameter $\underline{d}$ transforms Γ into the arc Γ^{**}. Obviously

$$L_{-d}L_d\Gamma = \Gamma. \tag{3}$$

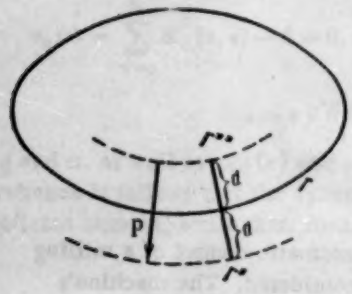

Fig. 1

Fig. 2

Expression (3) plays an important part in the subsequent discussion since the choice of step and the method of spot control of computation are based on it.

We note that because of the existence of corner points on the piecewise continuous curve Γ (Fig. 2), separate equidistant curves have to be connected by junction arcs which can be chosen by different methods.

Piecewise continuous closed curves Γ will be subsequently investigated. Counterclockwise travel along a curve will be considered positive. Parameters of the equidistants will be denoted by asterisks. Thus, for example, Γ is the principal curve, Γ^* is an equidistant, $\underline{x}$ and $\underline{y}$ are coordinates of Γ, x^* and y^* are coordinates of the equidistant, etc.

It is easy to see that the equation of the external equidistant can be written in the form

$$x^* = x \pm \frac{y'd}{\sqrt{1+(y')^2}}, \quad y^* = y \mp \frac{d}{\sqrt{1+(y')^2}}. \tag{4}$$

The upper signs are chosen if the projection of the arc Γ onto the $\underline{x}$ axis is positive; the lower signs are taken in the opposite case. The derivative is denoted by y'.

Usually Γ is given in the form of tabulated coordinates (x_s, y_s). Expressions (4) show that when the curve Γ is specified by a table of coordinates, the determination of coordinates (x_s^*, y_s^*) of the curve Γ^* requires the use of numerical differentiation.

For acceptable accuracy the curve Γ is broken up into sections with $\left|\frac{dy}{dx}\right| \leq 1$, where $\underline{x}$ is chosen as the independent variable, and sections with $\left|\frac{dy}{dx}\right| > 1$, where $\underline{y}$ is taken as the independent variable and hence $\left|\frac{dx}{dy}\right| < 1$. Input data concerning the curve are fed into the digital computer by separate units which are supplied with the characteristics of the independent variable. In this way, in numerical differentiation one has to deal with derivatives the moduli of which do not exceed unity.

The ordinates of curve Γ are specified at equidistant points with interval $\underline{h}$. The determination of the derivative y'_s at the point $x_s = x_0 + sh$ can be carried out by means of the familiar difference equation

$$y'_s = \frac{1}{h}\left[\frac{3}{4}(y_{s+1} - y_{s-1}) - \frac{3}{20}(y_{s+2} - y_{s-2}) + \frac{1}{60}(y_{s+3} - y_{s-3})\right]. \tag{5}$$

The remainder term of this formula is equal to $\frac{1}{140} h^6 y^{(7)}$ and, for a sufficiently small interval h is very small.

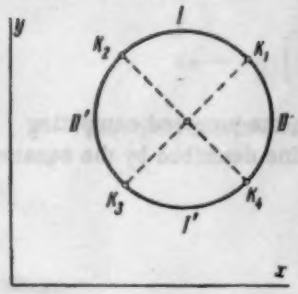

Fig. 3

The interval is chosen on an SDC computer by the method of successive approximations. Some zero approximation h_0 is specified. For a point A on the principal curve A* and then $L_{-d}A^*$ are calculated. The distance between A and $L_{-d}A^*$ for a correctly chosen step must not exceed the given measure of accuracy ϵ. If the distance turns out greater than ϵ, then h_1 is taken to be $\left[\frac{1}{4} h_0\right]$, the calculation is repeated, etc.

The distance between the points A and $L_{-d}A^*$ is calculated on the SDC from time to time as a check on the accuracy of the calculation, and thus serves as a means of spot control. Integral control can be achieved by comparing the coordinates of the starting and final points on the curve Γ^*, since their difference must lie within the limits of the specified accuracy ϵ.

In the proposed method, circles with radius d (where d is the radius of the cutter) are used as the junction arcs of adjacent equidistants. It is not expedient to calculate directly the ordinates of the circle with center at the point (a, b) using the equation

$$y = b \pm d \sqrt{1 - \left(\frac{x-a}{d}\right)^2} \tag{6}$$

because of the difficulties associated with the nonsingle-valuedness and the loss of accuracy in the regions of large derivatives. Consequently, it is convenient to divide the circle into quadrants I, I', II, and II' by means of the points K_1, K_2, K_3, and K_4 (Fig. 3). The diameter K_2K_1 forms the angle $\frac{\pi}{4}$ with the x axis, while the diameter K_4K_2 forms the angle $-\frac{\pi}{4}$; the diameters are mutually perpendicular. In quadrants I and I' x is the independent variable, while y is in quadrants II and II'.

Making use of the approximations of function (6) by the best polynomial P_n, we have:

quadrant I

$$\frac{y-b}{d} = P_n\left[\left(\frac{x-a}{d}\right)^2\right], \tag{7}$$

quadrant I'

$$\frac{y-b}{d} = -P_n\left[\left(\frac{x-a}{d}\right)^2\right], \tag{8}$$

quadrant II

$$\frac{x-a}{d} = P_n\left[\left(\frac{y-b}{d}\right)^2\right], \tag{9}$$

quadrant II'

$$\frac{x-a}{d} = -P_n\left[\left(\frac{y-b}{d}\right)^2\right]. \tag{10}$$

Taking n = 4, we obtain

$$P_4(z) = 0{,}999 \cdot 459 \cdot 91 - 0{,}482 \cdot 935 \cdot 85z^2 - 0{,}201 \cdot 380 \cdot 46z^4, \tag{11}$$

where for Eqs. (7) and (8) we must take $z = \frac{x-a}{d}$, while for Eqs. (9) and (10) $z = \frac{y-b}{d}$.

In the calculation of junction arc B'B (Fig. 2) on an SDC, first the location of the given curve with respect to the quarters is investigated, the initial independent variable is determined, and then calculations based on formulas (7) - (10) are carried out. The obtained coordinates are compared simultaneously with the coordinates of the points K_i to ensure a well-timed transition from one polynomial to another. In the last quarter the calculated coordinates are compared with the final coordinates of point B' to ensure a timely stopping of the calculation of the junction arc.

2. Correction of the Equidistant

It should be noted that in many particular cases which are of practical interest, the proposed computing system is considerably simplified. Thus, for example, corresponding to the straight line described by the equation $y = kx + b$ is, from Eq. (4), the equidistant straight line

$$x^* = x \pm \frac{kd}{\sqrt{1+k^2}}, \quad y^* = kx + b \mp \frac{d}{\sqrt{1+k^2}}.$$

In the same way, corresponding to the circle of radius R is the equidistant consisting of the concentric circle of radius $R \pm d$. Equidistants to general second-order curves are not second-order curves, but their calculation is simple in comparison with the general case.

Consequently, it is logical to calculate those milling curves which frequently occur in practice, and which consist of pieces of straight lines and circular and parabolic arcs, by means of a simplified program which can be carried out on an SDC independently of the general program.

An experimental program of this type was composed for the "Ural" universal machine and was tried out. In considering this particular case one can introduce over-all simplification in the circuit of the SDC.

Let us examine the question of correcting the equidistant. Small deviations in the radius of the cutter Δd from the initial value $\underline{d}$ due to wearing, temperature, etc., are always possible.

When it is necessary to introduce a correction, the adjusted values of the coordinates x^*_{sn} and y^*_{sn} are found from the expression

$$x^*_{sn} = x^*_s \pm \frac{\Delta d y'_s}{\sqrt{1+(y'_s)^2}},$$

$$y^*_{sn} = y^*_s \mp \frac{\Delta d}{\sqrt{1+(y'_s)^2}}.$$

The upper signs are chosen when $\mathrm{pr}_X \Gamma > 0$, the lower ones when $\mathrm{pr}_X \Gamma < 0$.

The quantities $\frac{y'_s}{\sqrt{1+(y'_s)^2}}$ and $\frac{1}{\sqrt{1+(y'_s)^2}}$ are known from the calculation of Γ^* and, consequently, the introduction of a correction merely involves multiplying these quantities by Δd and adding the products to x^*_s and y^*_s

3. Processing of Data for an AU

In the preceeding discussion the necessity of simplifying the mathematical operations of the AU unit which produces the control pulses was stressed. This can be achieved by limiting the mathematical functions of the AU to the successive calculation of the values of polynomials.

It is known that in the calculation of a series of values of a polynomial $P_n(x) = \sum_{k=1}^{n} a_k x^k$ the operations of multiplication and raising to a power can be avoided if difference methods are used. Choosing the step $\underline{h}$, the ordinate y_0, and the differences $\Delta y_0, \Delta^2 y_0, \ldots, \Delta^n y_0$, we are able to calculate consecutively ordinates of the polynomial $y = P_n(x)$ at the points $x = x_0 + ih$ $(i = 1, 2, \ldots)$ making use of the operation of addition only.

Naturally, by simplifying the AU we impose on the SDC digital computer the added functions of approximating the ordinates of equidistant curves by polynomials. As an effective means of approximating an equidistant curve in the interval $a \leq x \leq b$ we make use of n-th degree polynomials $Q_n(x)$ defined by the values of the ordinates at n+2 generalized Tchebycheff modes:

$$x_k = \frac{1}{2}\left[(a+b) + (b-a)\cos\frac{2k-1}{2(n+2)}\pi\right] \quad (k = 1, 2, \ldots, n+2).$$

These polynomials are of the form

$$Q_n(x) = \sum_{k=1}^{n+2} [y_k + (-1)^{k+1} M] \frac{A(x)}{(x-x_k) A'(x_k)},$$

where

$$M = \frac{\sum_{k=1}^{n+2} \frac{y_k}{A'(x_k)}}{\sum_{k=1}^{n+2} \frac{(-1)^k}{A'(x_k)}}, \quad A(x) = (x-x_1)(x-x_2)\ldots(x-x_{n+2}).$$

The approximating polynomial $Q_n(x)$ must satisfy, in the interval $a \leq x \leq b$, the inequality

$$|f(x) - Q_n(x)| < \varepsilon,$$

where $y = f(x)$ is the equation of the equidistant curve, while ϵ is the specified accuracy. Then there arises the problem of selecting the maximum permissible step, corresponding to the linear displacement of the cutter from node x_k to x_{k+1} $(k = 1, 2, \ldots)$. The smallest step h_{min} is specified and we investigate the possibility of increasing it.

To do this we determine constants $d_1, d_2, \ldots, d_p$, satisfying the inequalities $a < d_1 < d_2 < \ldots < d_p \leq b$. The values of d_s $(s = 1, 2, \ldots, p)$ are found successively. If $d_1, d_2, \ldots, d_s$ are known, then to find d_{s+1} we examine the polynomial

$$N(\xi) = \frac{1}{2}\left[(d_s - \xi) Q'_n(\xi) + Q_n(\xi) - Q_n(d_s)\right],$$

which changes monotonously in the vicinity of d_s. First we determine ξ^* from the condition $2|N(\xi^*)| = \epsilon$. Once ξ^* is found, to find d_{s+1} it is necessary only to solve the algebraic equation. [$Q'_n(x)$ is the derivative of the polynomial $Q_n(x)$].

$$d_{s+1} Q'_n(\xi^*) - Q_n(d_{s+1}) = d_s Q'_n(\xi^*) - Q_n(d_s).$$

The smallest of the numbers $d_1 - d_1, d_2 - d_1, \ldots, b - d_p$ is chosen as the new step in the interval $a \leq x \leq b$.

SUMMARY

Problems were programmed and solved on the universal electronic computer "Ural" to clarify the mathematical possibilities of the proposed method.

The "Ural" computer solved a trial problem having an outline consisting of 15 arcs in 1 min 48 sec. This involved the use of 678 short memory locations in the operational storage.

Analysis of data from experimental programming shows that it is expedient to have an SDC with a fixed period, operating with a 28 bit binary code, having an operational storage with a capacity of about 4000 codes, an auxiliary storage with a capacity of 10^6 codes, and an operating speed of 500 oper/sec.

An SDC computer with such parameters is capable of yielding, over a period of 8 hours of operation without breakdown, about 100 perforated tapes, i.e., solving about 100 problems. This time, of course, does not include preparation of data, checking and correction of tapes.

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SYNTHESIZING RELAY SYSTEMS FOR DETERMINING PERMUTATIONS

I. S. Danilyuk

(L'vov)

The operating conditions for relay circuits designed to determine the order in which n contacts close are formulated as n^2 Boolean functions; the corresponding circuits are also given. The operations of these circuits are studied, and it is shown that the total number of relays can be reduced, so that new, more economical contactor circuits result.

Problems of the number of openings (or closings) of sets of relay contacts often arise in telemechanics, relay automatics and computing. Devices designed to solve these problems are best built up from two-position contactors.

Some aspects of such devices are considered here(by 'permutations' we mean the sequence of operations, taken in any order, of a given set of contacts). The design method used for multicycle circuits is based on a graphical representation of the operating conditions of the separate relays [1, 2].

1. Let us design a relay circuit to determine the permutations of two units. The input relays, whose drive coils are external to the circuit, are denoted by A_1 and A_2, while their make contacts are denoted by a_1 and a_2. We assume that each input relay has only one make contact. This case is commonly found when a relay is used as a sensing element which, for power or other reasons, cannot accommodate many contacts. Electronic devices with relay-type action also fall in this class. It is also assumed that we always have $t_c \geq t_i$, where t_c is the time for which the contact on the input relay is closed, and t_i is the time some intermediate relay takes to operate. Sequential operation of A_1 and A_2 gives us the permutations A_1A_2 and A_2A_1.

The number of effector relays equals the number of permutations (here two). This circuit is a multicycle one, and so there must be intermediate relays; the number of such relays can be established as follows. Either A_1 or A_2 may operate first in both of the permutations, A_1A_2 and A_2A_1. There must be two intermediate relays per input relay; of these, one operates from the first operation of its input relay, and the other from the second. The total number of intermediate relays is thus four. A simple argument shows that a circuit with n inputs has n^2 intermediate relays, divided into n sets of n each. We denote these relays by X_{ij} ($i, j = 1, 2, 3, \ldots, n$), where i is the number of the set, and j is the number of the relay within a set. The relays in the first set will thus be X_{11} and X_{12}, and those in the second set X_{21} and X_{22}.

The operating conditions may be described in words as follows:

1) in the initial state all the first relays in the sets must have their circuits in the ready-to-operate state, whereas the second relays must not;

2) the first relay to operate must switch the circuits of the relays in the second set in such a way that the first relay cannot be energized, whereas the second can;

3) all relays which have operated must be held on by their own contacts.

Tables 1 and 2 show the operations for A_1A_2 and A_2A_1, respectively.

TABLE 1

Relay		Cycle 0	1	2	3	4	5	6
A_1	1		—	—	—	—	—	—
A_2	2					—	—	—
X_{11}	4			—	—	—	—	—
X_{12}	8							
X_{21}	16							
X_{22}	32						—	—
Number of combinations		0	1	5	4	6	38	36

TABLE 2

Relay		Cycle 0	1	2	3	4	5	6
A_1	1					—	—	—
A_2	2	—	—	—	—	—	—	—
X_{11}	4							
X_{12}	8						—	—
X_{21}	16			—	—	—	—	—
X_{22}	32							
Number of combinations		0	2	18	16	17	25	24

Each unit in the circuit is represented by a row, in which a thick line denotes cycles which define that unit's energized state, and to which unit is assigned a number corresponding to its rank in the binary system. The broken lines denote cycles in which the state is without significance, i.e., the relay can be energized or not. The number in the bottom row is the number of combinations in which the energized state occurs. No numbers are duplicated in this last row; the circuit can thus be realized with the number of relays used [3].

We can now set down the conductances of the contact circuits that act on the intermediate relays as four Boolean functions

$$\begin{aligned} f_{11} &= \bar{x}_{21}(a_1 + x_{11} + x_{12}), \\ f_{12} &= x_{21}(a_1 + x_{11} + x_{12}), \\ f_{21} &= \bar{x}_{11}(a_2 + x_{21} + x_{22}), \\ f_{22} &= x_{11}(a_2 + x_{21} + x_{22}). \end{aligned} \tag{1}$$

Now (1) is so simple that we can construct the corresponding circuit directly. Figure 1 shows this circuit. Here a_1 and a_2 are the make contacts on the input relays, which lie outside the circuit, and Y_1 and Y_2 are the drive coils of the effector relays. Permutation A_1A_2 corresponds to x_{11} and x_{22} being closed in the circuit of Y_1, and A_2A_1 to x_{21} and x_{12} in Y_2's circuit.

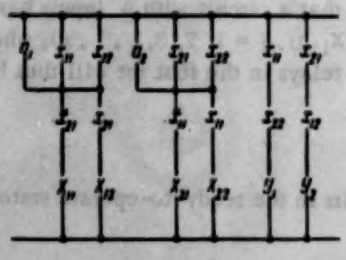

Fig. 1

Analogous arguments (not given here) enable us to construct a circuit for deriving the permutations of three units, A_1, A_2 and A_3. Figure 2 shows this circuit; it consists of three sets of intermediate relays, with three in each set. Each set is controlled by one make contact on an input relay. The number of permutations (six) gives us six distinct circuits for the effectors Y_1-Y_6.

Figures 1 and 2 can be analyzed to yield the general features of such circuits, where the inputs are n in number. These features are as follows:

1) the circuit of the first relay in any set contains one break contact from each of the other first relays, all of which contacts are joined in series;

2) the circuit of the second relay in any set contains an inversion of the series contact circuit of the first relay in the same set, and break contacts from all the other second relays, all joined in series;

3) the circuit of any intermediate relay (up to the last-but-one inclusive) is designed on the principle given for the second relay, i.e., by inverting the series break-contact circuit of the preceding relay in the same set, and with break contacts from all the other relays of the same number;

4) the circuit of any last relay contains only an inversion of the series circuit of the preceding relay in that set;

5) each set of intermediate relays is controlled by a make contact on an input relay, and all the relays are held on by their own contacts.

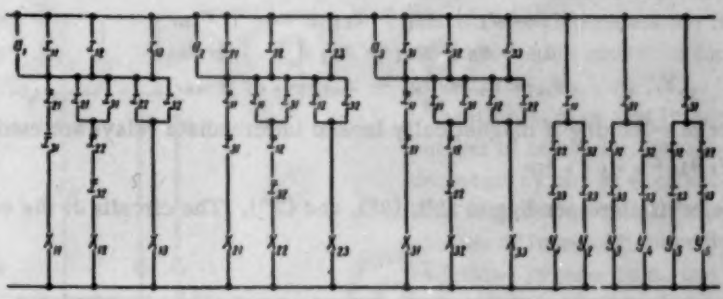

Fig. 2

This set of statements can be represented as n^2 Boolean functions of the independent variables a_i and of the variables x_{ij} ($i, j = 1, 2, 3, \ldots, n$). The equations for the contact circuits of the 1st, k-th, and n-th sets ($1 < k < n$) are

a) first set

$$f_{11} = \bar{x}_{21}\bar{x}_{31}\ldots\bar{x}_{n1}\sigma_1 = \sigma_1 \prod_{i=1}^{n} \bar{x}_{i1},$$

$$f_{12} = (x_{21} + x_{31} + \ldots + x_{n1})(\bar{x}_{22}\bar{x}_{32}\ldots\bar{x}_{n2})\,\sigma_1 = \sigma_1 \sum_{i=1}^{n} x_{i1} \prod_{i=1}^{n} \bar{x}_{i2}, \qquad (2')$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$f_{1n} = (x_{2(n-1)} + x_{3(n-1)} + \ldots + x_{n(n-1)})\,\sigma_1 = \sigma_1 \sum_{i=1}^{n} x_{i(n-1)} \quad \text{for} \quad i \neq 1;$$

b) k-th set

$$f_{k_1} = \sigma_k \prod_{i=1}^{n} \bar{x}_{i1},$$

$$f_{k2} = \sigma_k \sum_{i=1}^{n} x_{i1} \prod_{i=1}^{n} \bar{x}_{i2}, \quad \text{for} \quad i \neq k); \qquad (2'')$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$f_{kn} = \sigma_k \sum_{i=1}^{n} x_{i(n-1)}$$

c) n-th set

$$f_{n1} = \sigma_n \prod_{i=1}^{n} \bar{x}_{i1},$$

$$f_{n2} = \sigma_n \sum_{i=1}^{n} x_{i1} \prod_{i=1}^{n} \bar{x}_{i2}, \quad \text{for} \quad i \neq n) \tag{2'''}$$

. . . . . .

$$f_{nn} = \sigma_n \sum_{=1}^{n} x_{i(n-1)}.$$

The quantities σ_1, σ_k and σ_n in (2'), (2") and (2"') are the common parts of the 1st, k-th, and n-th sets, and consist of hold-on contacts joined in parallel with one another and with the make contact on the input relay:

$$\begin{aligned} \sigma_1 &= a_1 + x_{11} + x_{12} + \ldots + x_{1n}, \\ \sigma_k &= a_k + x_{k1} + x_{k2} + \ldots + x_{kn}, \\ \sigma_n &= a_n + x_{n1} + x_{n2} + \ldots + x_{nn}. \end{aligned} \tag{3}$$

There is no need for self-locking if magnetically locked intermediate relays are used: then $\sigma_1, \sigma_k, \ldots, \sigma_n$ become, respectively, $a_1, a_2, \ldots, a_n$.

Figure 3 shows the circuit corresponding to (2'), (2"), and (2"'). The circuits of the effectors have no point of interest, and so are omitted.

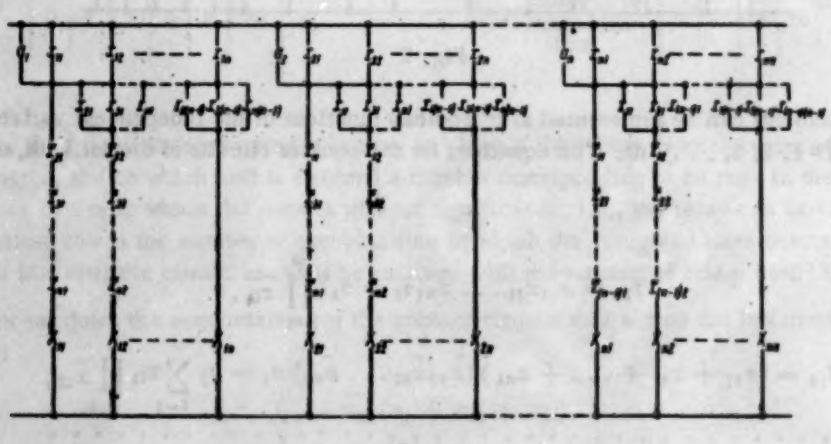

Fig. 3

These circuits show that the contacts on the intermediate relays can be divided into ones that energize the effectors, and ones that control the sequential working of the relays themselves.

The number of contacts of the first type, per relay, does not exceed $(n-1)!$ We can reduce this number to $A_n^j = n(n-1)\ldots(n-j+1)$, where $j = 1, 2, \ldots, n$, by simply combining separate contacts on any given relay. The total number of contacts for all relays is then $\sum_{j=1}^{n} A_n^j$. The number of contacts in the effector circuits can be reduced, e.g., by using rectifiers and by putting the contacts on both sides of a relay.

Each relay X_{ij} $(j \neq n)$ has $2(n-1)+1$ contacts, all of which are needed to sequence the operations. All the X_{in} relays have only one hold-on contact each. In all $n^2(2n-3)+2n$ contacts are needed to sequence the operations. The total number of contacts on the intermediate relays is

$$K_n' = n^2(2n-3) + 2n + \sum_{j=1}^{n} A_n^j. \tag{4}$$

In all these arguments we have assumed that the number of contacts available on an intermediate relay is unlimited. Some relays may in practice have to be duplicated.

2. Let us consider in detail the operations during some one cycle, meaning, let us examine some one permutation. The first input causes one of the n first intermediate relays to operate. Now any input can enter only once into any combination, so the set of relays that has operated is debarred from further operations during that cycle. Then the second input causes one of the $(n-1)$ second relays to operate. This set of relays is then debarred from further operations during that cycle. In general, the number of free (unoperated) sets diminishes by unity as each input is energized. The last input can operate only one relay, namely the last one in the last set remaining. We can simplify the argument by entering all the relays in a square table, with the i-th set of relays taking the i-th row, and all the relays of number j the j-th column. The table resembles the matrices of linear algebra. Any permutation is derived by passing in sequence from a relay in the 1st column to a relay in the n-th column. When relay X_{ij} has operated, the whole of set i and group j (i.e., the relays in the i-th row and and j-th column) are excluded from operating further. This form of representation is convenient also when we consider the equations for the circuits of the effectors $Y_1, Y_2, \ldots, Y_{n!}$. Only n relays, out of the n^2 available, take part in any one permutation. The number of relays left capable of operating in any set decreases by one as each successive input is energized. This feature shows that we may be able to reduce the number of intermediate relays somewhat. We proceed as follows in order to do this reduction. We divide the total number of relays into n sets, in accordance with the sequence of operation. The first input we put in correspondence with a set of n relays, the second with a set of $(n-1)$ relays, the third with $(n-2)$, etc., down to the last input, which will have only one relay. The drive coils on the intermediate relays we denote, as before, by double subscripts. The first subscript denotes the number of the set, the second the number of the relay within the set. The total number of relays will equal the sum of the first n natural numbers, i.e.,

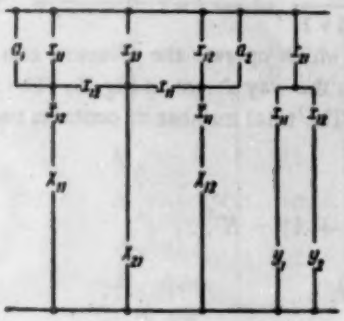

Fig. 4

$$\frac{n(n+1)}{2}.$$

Figure 4 shows a circuit to effect this system for two inputs, A_1 and A_2. The circuit contains three intermediate relays, X_{11}, X_{12}, and X_{21}. X_{11} and X_{12} operate from the first input, and X_{21} from the second. X_{11} and X_{12} have their circuits prepared for operation in the initial state. If now the first input in the permutation is A_1, then X_{11} operates when a_1 closes, prepares X_{21}, breaks the prepared circuit of X_{12}, and prepares the circuit of effector Y_1. X_{11} also holds on via its own contact. The second input, A_2, operates X_{21}, which closes Y_1's circuit. The operations are analogous if the permutation is A_2A_1. The circuit of Y_2 is then closed.

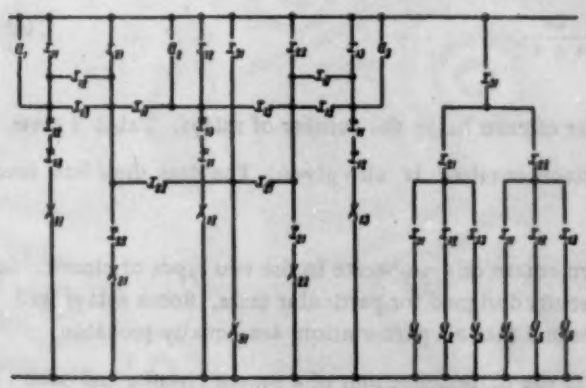

Fig. 5

Figure 5 shows a circuit for three inputs. There are six intermediate relays in the circuit, of which the part containing X_{21}, X_{22}, and X_{31} resembles Fig. 4. The set X_{11}, X_{12}, and X_{13} is assigned to the first element in the permutation. Consider the working for permutation $A_1A_2A_3$. Relay X_{11} operates when a_1 closes, thereby disconnects X_{12} and X_{13} from a_2 and a_3, and connects in place of these two relays the set X_{21}, X_{22}, and X_{31}. Relay X_{21} operates when a_2 closes, thereby disconnects X_{22} from a_3, and connects instead X_{31}, which operates when a_3 closes. The relays are all self-locking, and close the circuit of effector Y_1.

The reduced circuits have bridge-connected contacts. We shall use the arbitrary names 'square' for circuits of the types of Figs. 1, 2, and 3, and 'triangular' for those of the types of Figs. 4 and 5.

If we compare triangular circuits for different values of n, we find that, as with the square circuits, the systems have much in common. The circuit with n inputs can be represented as the circuit with $(n-1)$ inputs joined to n relays of the first set. Thus, the circuit for $n = 3$ contains the circuit for $n = 2$ and three first-set relays; the circuit for $n = 4$ includes the circuit for $n = 3$ and four first-set relays, etc. Any relay in the i-th group operates and disconnects all the other relays in its own group; connects the residual $(n-1)$ contacts on the inputs to the $(i+1)$ th set. The circuit of every relay in the i-th set contains break contacts from all the other relays in that set all joined in series, while the $(n-i)$ remaining inputs are connected to the $(i+1)$ th set via the make contacts of the relay that has operated. There are also control and accessory parts of the circuit, which contain, respectively, the n make contacts on the inputs, and $\frac{n(n+1)}{2}$ make contacts on the relays (by which they hold on). The number of contacts on an intermediate relay in a triangular circuit depends on the number of the set. The number of contacts needed to work the effectors is $\frac{n!}{n-i+1}$, where $i = 1, 2, \ldots, n$. This number can be reduced considerably in various ways. The number of contacts which operate the effectors can be reduced to $(n-1)!$ per intermediate relay by simply combining the contacts in the way shown in Fig. 5. The number of contacts per relay needed to ensure proper sequencing is $2(n-i)+1$. The total number of contacts needed for the effectors per relay when these methods are used is given by

$$K'_n = \sum_{i=1}^{n} (n-i+1)\,[(n-i)! + 2(n-i)+1] = K'_{n-1} + \\ + n(n-1)! + n(2n-1). \tag{5}$$

3. We now introduce the use factor for a relay; this factor is the ratio of the number of permutations to the number of intermediate relays:

$$\beta = \frac{N}{M}. \tag{6}$$

In square circuits

$$\beta' = \frac{n!}{n^2} = \frac{(n-1)!}{n}. \tag{7}$$

In triangular circuits

$$\beta'' = \frac{2n!}{n(n+1)} = \frac{2(n+1)!}{n+1}. \tag{8}$$

The quotient from dividing (8) by (7) shows how much more economical the triangular circuits are:

$$\frac{\beta''}{\beta'} = \frac{2n}{n+1}. \tag{9}$$

If n is large, $\frac{\beta''}{\beta'}$ is nearly two, i.e., the triangular circuits halve the number of relays. Table 3 gives β', β'' and $\frac{\beta''}{\beta'}$ for n from 2 to 7. The number of contacts required is also given. The data show how much better the triangular circuits are.

Let us compare the frequencies with which the intermediate relays operate in the two types of circuit. Some permutations may occur more often than others in real circuits designed for particular tasks. Some relays will then operate more often than others. For simplicity we assume that all permutations are equally probable.

The intermediate relays will all be equally loaded in the n! permutations of a square circuit, and each relay will operate $(n-1)!$ times. The simplified circuits give a different pattern. The a priori probability of a relay's operating in a triangular circuit is $\frac{1}{n-i+1}$, where i is the set number, and so the relays in set 1

will operate $\frac{n!}{n}$ times during the n! permutations, the set 2 relays $\frac{n!}{(n-1)}$ times, the set 3 relays $\frac{n!}{(n-2)}$ times, etc; the last set operates n! times, because this relay has to operate. The total number of relay operations for all permutations of the n inputs is n! n. The relays thus have to operate somewhat more frequently in the simplified circuits. This more frequent operation has in practice no effect on the reliability. The reduced numbers of relays and of contacts increase the reliability substantially.

TABLE 3

n	2	3	4	5	6	7
β'	0.5	0.67	1.5	4.8	20	103
β''	0.67	1	2.4	8	34.3	180
$\frac{\beta''}{\beta'}$	1.33	1.5	1.6	1.67	1.72	1.75
K'_n	12	48	152	510	2280	14252
K''_n	10	31	83	248	1034	6165
$\frac{K'_n}{K''_n}$	1.2	1.55	1.88	2.05	2.2	2.31

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RELATIONS BETWEEN THE PARAMETERS OF RADIATION RELAYS

A. G. Vasil'ev and K. S. Klempner

(Kharkov)

The transients in radiation relays are considered. Relations are given that relate the response time to the time-constant to the operate threshold, and to the counting rate. Test data are given that confirm the deductions.

Several papers have appeared on the uses of radiation relays in automatics (see [1] for instance). The relations between the main parameters of such relays have to be deduced.

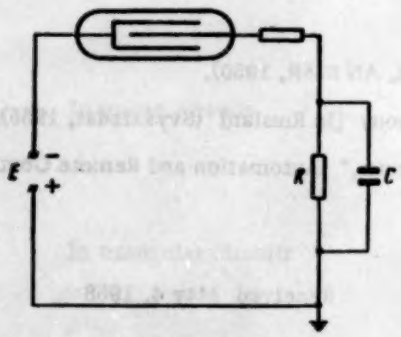

Fig. 1. The counter circuit.

All existing designs can be related to the operations of a geiger counter connected to a load in the form of an integrating circuit (Fig. 1).

We assume that the applied voltage E is constant, and that the counting rate does not exceed 100 counts/sec. These conditions are typical of those found in gamma relays, and are such that we can assume that the size and shape of the counter pulses are fixed, because the size and shape are defined by the quenching mechanism in the counter, and not by the external circuit, since the integrating time is very much longer than the duration of a single discharge [2]. We may also assume that the external impedance is independent of the shape and size of the pulses. The voltage pulse appearing across the load is determined by the current pulse, by the capacitance C*, and by the resistance R.

If R is large, the current i all flows to the capacitance and the voltage V_0 produced by one pulse is given by

$$V_0 = \frac{1}{C}\int_0^{t_0} i\,dt = \frac{Q}{C}, \tag{1}$$

where t_0 is the pulse length.

The time t_1 between two current pulses is given by

$$t_1 = \frac{1 - n_1 t_0}{n_1}, \tag{2}$$

where n_1 is the count rate when the variable absorber is absent.

* The capacitance to earth in the wire, and in all leads, is neglected.

Now $t_0 \approx 10^{-4}$ sec, and n_1 is from 10 to 10^2 counts/ sec, and so we take that $t_1 \approx 1/n_1$.

We call the state when there is no absorber, state 1. We have so far neglected the fluctuations in the mean count rates.

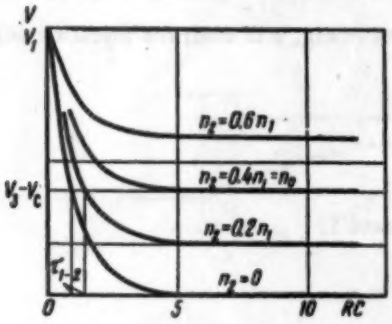

Fig. 2. The changes in potential caused by removing an absorber.

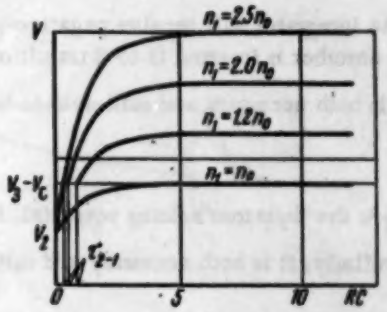

Fig. 3. The changes in potential caused by inserting an absorber.

The potential difference across the capacitor after the m-th pulse ($m \gg 1$) will be

$$V_m = \frac{Q}{C} \sum_{k=0}^{m} e^{-\frac{kt_1}{RC}} . \tag{3}$$

Let the time from the start to the instant when the m-th pulse arrives be $t = (m-1)t_1$; then using (2), we find the solution to (3) as

$$V_m = QRn_1\left(1 - e^{-\frac{t}{RC}}\right). \tag{4}$$

The steady p.d. developed across the capacitor after a long period is [3]

$$V_1 = QRn_1. \tag{5}$$

The above results can be used to show that, when the relay transfers to state 2, as an absorber is inserted between the source and the counter, the p.d. will vary as

$$V_{1-2} = QRn_2 + QR(n_1 - n_2)e^{-\frac{t}{RC}}, \tag{6}$$

where, in this case, t is the time since the absorber was inserted.

The steady p.d. in state 2 is [3] given by

$$V_2 = QRn_2. \tag{7}$$

$V_2 = 0$ if $n_2 = 0$.

Figure 2 shows the transient behavior in the potential during the 1 to 2 transition (for $Q > 0$). The time is shown in units of RC.

The p.d. will change in the 2 to 1 transition in accordance with

$$V_{2-1} = QRn_1 - QR(n_1 - n_2)e^{-\frac{t}{RC}}, \tag{8}$$

which becomes the same as (4) if $n_2 = 0$.

Figure 3 shows the transient behavior in the potential during the 2 to 1 transition (for Q > 0).

These results can be used to interpret the basic parameters of such relays.

The generality of the treatment is not restricted if the relay is taken as being of thyratron type.

The integrator will receive negative-going pulses if the circuit of Fig. 1 is used; the thyratron will fire when an absorber is inserted (1 to 2 transition).

It is both necessary and sufficient to have

$$V_1 < V_f , \tag{9}$$

where V_f is the thyratron's firing potential, for the relay to stay in state 1.

Similarly, it is both necessary and sufficient to have

$$V_2 > V_f \tag{10}$$

for the relay to stay in state 2.

Instead, the integrator can be connected to the cathode of the counter; positive-going pulses are then stored, and the thyratron fires when the absorber is removed (2 to 1 transition).

Which system is used depends on the main purpose of the relay (i.e., on what proportion of the time there is an absorber present).

The total counting rate that makes the grid potential equal to the firing potential (i.e., the potential at which the state changes) is termed the operate threshold, and is denoted by n_0 (counts/sec). This threshold defines the sensitivity; the lower the threshold the greater the sensitivity.

The threshold is determined by the difference between the dc bias and the firing voltage, if Q is negative, or by the same quantity, but with the sign reversed, if Q is positive. If no bias is used, the threshold is determined solely by the firing voltage. Then (9) shows that

$$n_0 = \frac{V_c - V_f}{QR} \ (\text{if} \quad Q<0), \qquad n_0 = \frac{V_f - V_c}{QR} \ (\text{if} \quad Q>0). \tag{11}$$

The sensitivity can be varied within wide limits.

We now consider the relay's response time. This time is the time taken to pass from state 1 to state 2, or conversely. The circuit is assumed to be that of Fig. 1.

We denote this time by τ, with subscripts to denote the direction of the transition, e.g., τ_{2-1}, τ_{1-2}. The transitions occur well before the changes in p.d. are completed (as in Figs. 2 and 3). In general τ_{1-2} and τ_{2-1} are not equal for given values of n_1, n_2, and n_0. The two times are found by solving (6) and (8), with QRn_0 inserted in place of V_{1-2} and V_{2-1}. The solutions are

$$\tau_{1-2} = RC \ln \frac{n_1 - n_2}{n_0 - n_2}, \tag{12}$$

$$\tau_{2-1} = RC \ln \frac{n_1 - n_2}{n_1 - n_0}. \tag{13}$$

Hence (12) and (13) relate the threshold, the response time, the count rate, and the integrating time.

Figures 2 and 3 show how τ_{1-2} and τ_{2-1} vary with n_1 and n_2.

Figure 4 shows oscillograms of the transients for two different count rates. The transients are exponential, as (6) and (8) show. The response times agree with those given by

$$\tau_{1-0} = RC \ln \frac{n_1}{n_0} \tag{14}$$

$$\tau_{0-1} = RC \ln \frac{n_1}{n_1 - n_0}. \qquad (15)$$

Here subscript 0 denotes the special case when state 2 has $n_2 = 0$.

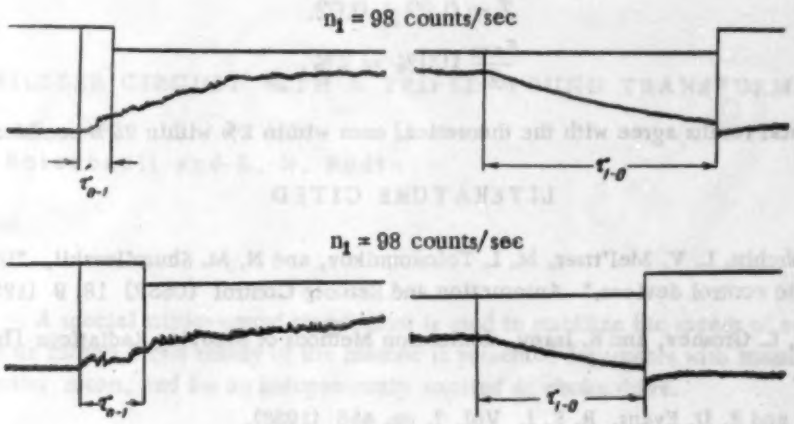

Fig. 4

We have verified the latter formula for various values of n_1, n_0, R, and C. We used a thyratron-type gamma relay [4]. The StS-1 counter was fixed in a holder at one end of an optical bench, along which slid a cobalt-60 source. A type B radiometer recorded n_1 for fixed source positions. The response time τ_{0-1} was measured with an electrical chronometer for those fixed source positions. Each measurement was repeated 15 times. The threshold n_0 was calculated from (16), which was derived by putting $V_m = QRn_0$ in (4):

$$n_0 = n_1 \left(1 - e^{-\frac{\tau_{0-1}}{RC}}\right). \qquad (16)$$

Series of readings were taken with various values of R, C, and n_0. RC and n_0 varied widely.

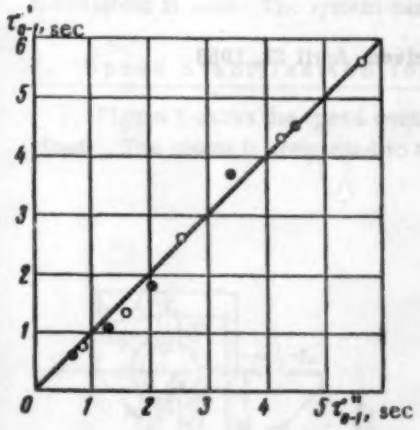

Fig. 5. Calculated and experimental response times ○ R = 6.6 MΩ, RC = = 2.64 sec; ● R = 6.6 MΩ, RC = = 3.30 sec.

The experimental response times (τ'_{0-1}) agreed with the ones calculated from (15) (τ''_{0-1}). In all, 105 values of τ'_{0-1} were used, and the corresponding τ''_{0-1} were calculated.

Figure 5 shows the τ_{0-1} values for two series. It is clear that theory and experiment agree well.

The results were worked up statistically [5]. The mean ratio of the theoretical and experimental values was calculated:

$$\bar{Z} = \frac{\sum_{i=1}^{n} Z_i}{n} = 0{,}99. \qquad (17)$$

The error in $\bar{Z}$, with a probability of 0.95, is

$$\varepsilon_{0.95} = S_{\bar{Z}} t_{0.95} = \sqrt{\frac{\sum_{i=1}^{n} (Z_i - \bar{Z})^2}{n(n-1)}}\, t_{0.95}, \qquad (18)$$

where $S_{\bar{Z}}$ is the standard deviation of the mean, n is the number of values of Z_i, $t_{0.95}$ is the parameter in Student's test [5] for a probability of 0.95 and $n > 30$.

If $S_{\bar{Z}} = 0.0099$ and $t_{0.95} = 1.96$, $\epsilon_{0.95} \approx 0.02$

Thus,

$$\bar{Z} = 0.99 \pm 0.02, \tag{19}$$

$$\frac{\epsilon_{0.95}}{\bar{Z}} 100\% = 2\%. \tag{20}$$

The experimental results agree with the theoretical ones within 2% within 95% confidence limits.

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A SPEED STABILIZER CIRCUIT WITH A TRIPLE-WOUND TRANSFORMER

O. B. Rosenbauli and R. N. Rodin

(Moscow)

A special triple-wound transformer is used to stabilize the speeds of ac and dc motors. The theory of the method is presented for motors with massive Shenfer rotors, and for an independently excited dc choke drive.

Ac and dc tachometers can be used to give velocity feedback in adjustable-speed drives; various types of feedback in terms of current or voltage can also be used.

The speed stabilizer considered here is for a choke-controlled dc drive; the feedback system includes current and voltage feedbacks in a parametric speed control for an ac motor, for which purpose a special triple-wound transformer is used. The system can be used with shunt-wound dc motors, and with three-phase ac motors.

1. Speed Stabilization for a Choke-Controlled dc Motor

Figure 1 shows the speed control circuits for a shunt-wound, choke-controlled dc motor (independently excited). The motor is connected to the line via a rectifier. A special triple-wound transformer with current, voltage and secondary windings is used. The first two windings are connected in opposition. The current coil carries the load current $I_{\sim}$, while the voltage coil is connected to the rectifier's ac terminals. The secondary coil supplies the stabilizer coil (connected in opposition to the control coil) on the choke via a rectifier.

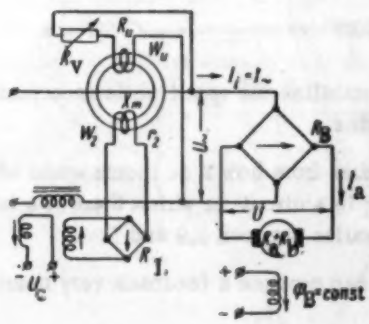

Fig. 1

Consider the current in the triple-wound transformer's secondary. We may take the rectifier and motor to behave as a resistive load (variable). We neglect leakage, losses in the steel, and magnetic saturation, and write the equations as

$$\dot{U}'_{\sim} = \dot{I}'_u R'_1 + \dot{E}'_u, \tag{1}$$

$$\dot{I}'_m = \dot{I}'_u - \dot{I}'_{\sim} - \dot{I}_2, \tag{2}$$

$$\dot{E}'_u = \dot{E}_2, \tag{3}$$

$$\dot{E}'_u = \dot{I}'_m j X'_m, \tag{4}$$

$$\dot{E}_2 = \dot{I}_2 R_2, \tag{5}$$

where R_1 is the voltage coil's resistance, I_u is the current in that coil, E_u is the resultant emf in that coil, E_2 is the resultant emf in the secondary, I_2 is the current in the secondary, of resistance R_2, I_m is the magnetizing current and X_m is the inductive impedance of the magnetizing circuit.

All the circuit parameters and the currents in (1) - (5), are referred to the secondary.

We solve (1) - (5) to give the secondary current as:

$$I_2 = \frac{U'_{\sim} - I'_{\sim} R'_1}{\sqrt{(R'_1 + R_2)^2 + \left(\frac{R_2 + R_1'}{X'_m}\right)^2}}. \tag{6}$$

We eliminate the reference to the secondary in the numerator in (6) and get

$$I_2 = \frac{U - \frac{W_1 R_1}{W_u} I_{\sim}}{\frac{W_u}{W_2}\sqrt{(R'_1 + R_2)^2 + \left(\frac{R_2 R'_1}{X'_m}\right)^2}} = k(U_{\sim} - k_1 I_{\sim}). \tag{7}$$

Here W_1 is the number of turns on the current coil, W_u is the same for the voltage coil, and W_2 is for the secondary;

$$k_1 = \frac{W_1 R_1}{W_u} \text{ and } k = \frac{W_2}{W_u\sqrt{(R'_1 + R_2)^2 + \left(\frac{R_2 R'_1}{X'_m}\right)^2}}$$

We now introduce the symbols I_a (armature current), U (voltage applied to motor), R_B (resistance of rectifier), and k_i and k_u (rectification coefficients for current and voltage, respectively); the rectifier's currents and voltages are then given by

$$I_a = I_{\sim} k_i, \tag{8}$$

$$U = U_{\sim} k_u - I_a R_B. \tag{9}$$

We combine (7), (8), and (9) to give the final expression for the secondary current as

$$I_2 = k_2(U - I_a k_3), \tag{10}$$

where

$$k_2 = \frac{k}{k_u}, \quad k_3 = \frac{k_1 k_u}{k_i} - R_B.$$

It is clear from (10) that the triple-wound transformer enables one to stabilize the speed while incorporating feedbacks with respect to load current and voltage, and all in one static device.

Now k_u will vary somewhat with the load on the motor. The cause arises from how a dc motor works when it is fed from an ac line via a rectifier. The rectifier is affected by working in a circuit in which there is a back emf. The changes in k_u are small, though. Within the working range, k_u varies between 0.9 and 1.

Thus, if we assume some value of k_u that lies within these limits, we can produce a feedback very nearly proportional to speed. This may be shown as follows.

The motor's equation is

$$U = cn\Phi_B + I_a(R_a + R_b). \tag{11}$$

We combine this with (8) and (9) to give

$$U_{\sim} - I_{\sim}\frac{k_i}{k_u}(R_B + R_a + R_b) = \frac{cn\Phi_B}{k_u}. \tag{12}$$

We put

$$\frac{k_i}{k_u}(R_n + R_a + R_b) = R_e \tag{13}$$

and

$$\frac{c\Phi_n}{k_u} = c_n, \tag{14}$$

and get the speed in the form

$$n = \frac{U_\sim - I_\sim R_e}{c_n}. \tag{15}$$

Comparing (7) and (15), we see that, if

$$k_1 = R_e \quad \text{or} \quad \frac{W_i R_i}{W_u} = \frac{k_i}{k_u}(R_n + R_a + R_b) = \text{const}$$

the triple-wound transformer in Fig. 1 acts as a static speed transducer.

We solve (7) with (15) to give $I_2 = k_\theta n$, where $k_\theta = c_n k$.

Thus, we can produce a feedback very nearly proportional to speed from one static unit by using the triple-wound transformer.

2. Speed Stabilization with Shenfer-rotor Motors

Figure 2 shows the speed control and stabilization system for a motor with a massive rotor. The motor is connected to the line via a three-phase choke. The triple-wound stabilizing transformer is similar to that in Fig. 1, except that an extra inductor X (an air-gap choke) is connected in the voltage coil's circuit. The current coil carries the load current $I_\sim$, and the voltage coil is connected to the line voltage $U_\sim$.

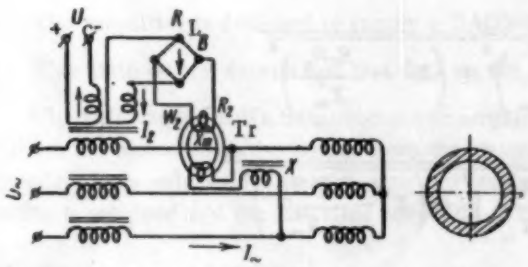

Fig. 2

We neglect leakage, etc., as before, and derive the current in the secondary. We use our previous notation, and write the equations for the transformer, allowing for the extra inductance in the voltage coil's circuit:

$$\dot{U}'_\sim = \dot{I}'_u R'_1 + \dot{I}'_u jX' + \dot{E}'_u, \tag{16}$$

$$\dot{I}'_m = \dot{I}'_u - \dot{I}'_\sim - \dot{I}'_2, \tag{17}$$

$$\dot{E}'_u = \dot{I}'_m jX'_m, \tag{18}$$

$$\dot{E}'_u = \dot{E}_2, \tag{19}$$

$$\dot{E}_2 = \dot{I}_2 R_2. \tag{20}$$

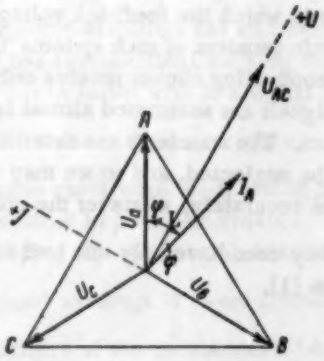

Fig. 3

All the parameters, and the voltages and currents, in (16) - (20) are referred to the secondary. Then (17) to (20) give us

$$\frac{I_2 R_2}{jX_m} = \dot{I}'_u - \dot{I}'_\sim - \dot{I}_2. \tag{21}$$

We take the line voltage vector as real axis, and express the load current as the sum of two projections:

$$\dot{I}'_\sim = I'_\sim \cos\varphi - jI'_\sim \sin\varphi. \tag{22}$$

The vector diagram (Fig. 3) shows that, if the transformer is connected as in Fig. 2, the angle φ can be found from

$$\varphi = \varphi_L - 30^\circ. \tag{23}$$

We combine (16), (19), (20), and (22) to give the secondary current as

$$I_2 = \frac{\sqrt{[U'_\sim - I'_\sim (R'_1 \cos\varphi + X' \sin\varphi)]^2 + I'^2_\sim (X' \cos\varphi - R'_1 \sin\varphi)^2}}{\sqrt{\left(R'_1 + R_2 + R_2 \frac{X'}{X'_m}\right)^2 + \left(X' - \frac{R_2 R'_1}{X'_m}\right)^2}}. \tag{24}$$

The power factor of this type of motor hardly varies at all with the load current [1]. Hence, if for the extra inductance we take

$$X' = \frac{R'_1 \sin\varphi}{\cos\varphi}, \tag{25}$$

we have (24) in the form

$$I_2 = \frac{U'_\sim - I'_\sim \frac{R_1}{\cos\varphi}}{\sqrt{\left(R'_1 + R_2 + R_2 \frac{X'}{X'_m}\right)^2 + \left(X' - \frac{R_2 R'_1}{X'_m}\right)^2}} = k'_2 U_\sim - I_\sim k'_3, \tag{26}$$

where

$$k_2' = \frac{W_2}{W_1 \sqrt{\left(R'_1 + R_2 + R_2 \frac{X'}{X'_m}\right)^2 + \left(X' - \frac{R_2 R'_1}{X'_m}\right)^2}},$$

$$k_3' = \frac{R'_1 W_1}{W_2 \cos\varphi \sqrt{\left(R'_1 + R_2 + R_2 \frac{X'}{X'_m}\right)^2 + \left(X' - \frac{R_2 R'_1}{X'_m}\right)^2}}.$$

Hence (26) shows that the circuit of Fig. 2 provides voltage and current feedback in a parametric speed control system for this type of ac motor.

This system has much in common with speed-stabilizing circuits that use normal current and voltage transformers with rectifiers in the secondary circuits, and with circuits in which the feedback voltage is taken directly from the motor terminals. Several works deal with the dynamic response of such systems (see [1], for instance), which is good. Our system is also good, in that the power-controlling chokes receive only the resultant negative feedback signal, and that the current and voltage feedback signals are summated almost instantly by the triple-wound transformer, since this summation occurs in the ac section. The transients are determined largely by the electromechanical processes, and electromagnetic effects can be neglected, and so we may conclude that the signal from the triple-wound transformer reflects the motor's speed accurately, whatever the rate of change.

The power chokes can also be smaller in this system, because they need have only one coil for the resultant feedback, instead of the two large coils required in the ordinary system [1].

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A FAST TRANSISTORIZED MAGNETIC PUSH-PULL AMPLIFIER FOR A SERVO DRIVE

V. S. Volodin, E. D. Larin, M. A. Rozenblat, and G. V. Subbotina

(Moscow)

It is shown that a high-speed amplifier for a servo drive can be made by combining magnetic and transistor amplifiers.

Test data are given for such an amplifier designed to control a two-phase asynchronous motor; the transient duration is 1-1.5 cycles at the supply frequency, and the power gain is 1.5×10^6.

Magnetic and transistor amplifying stages are best combined in automatic control amplifiers. The authors have developed a full-wave power amplifier of short response time in which such stages are combined; the amplifier has first-class technical and economic features.

The amplifier is designed to supply a DAD2-350/50 hollow-rotor asynchronous two-phase motor.

The main design details and test data on the amplifier are given here.

The first stages are of a transistorized ac amplifier, which has a very short rise time. The magnetic power amplifier is connected to the output from the phase-sensitive detector. The combination of transistor stages with magnetic stages reduces the weight, size and cost relative to a multistage magnetic amplifier, while the reliability is retained and the rise time reduced.

The Magnetic Amplifier

Power transformers were eliminated by using a push-pull bridge circuit fed directly from the 220 v, 50 cps line.

The magnetic amplifier has an internal feedback, to ensure rapid response at high gain (Fig. 1). Each of the four cores has two identical working windings W' and W", which are connected in opposite arms of the bridge. Each winding has in series with it a metal rectifier, which produces the internal feedback.

Figure 1a shows the power circuits, and Fig. 1b the control and bias circuits.

High-grade cores were used, and so the initial currents in the bridge arms were large, which caused overheating and resulted in poor performance. These initial currents were reduced and the optimum operating conditions obtained by applying ac bias directly from the line; resistance R_{CM} sets this bias.

The control windings W were connected in series to the output of the transistorized amplifier.

The functions of the resistances R (shown by broken lines in Fig. 1) are detailed below.

Wound toroidal cores of É-310 cold-rolled steel strip 0.08 mm thick were used, with the turns insulated by the oxide coatings produced by the annealing. The annealed cores had hysteresis loops that were nearly rectangular.

Each core weighed 4075 g, and had an internal diameter of 82 mm, an outside diameter of 138 mm, and a height of 70 mm.

Table 1 gives the winding parameters.

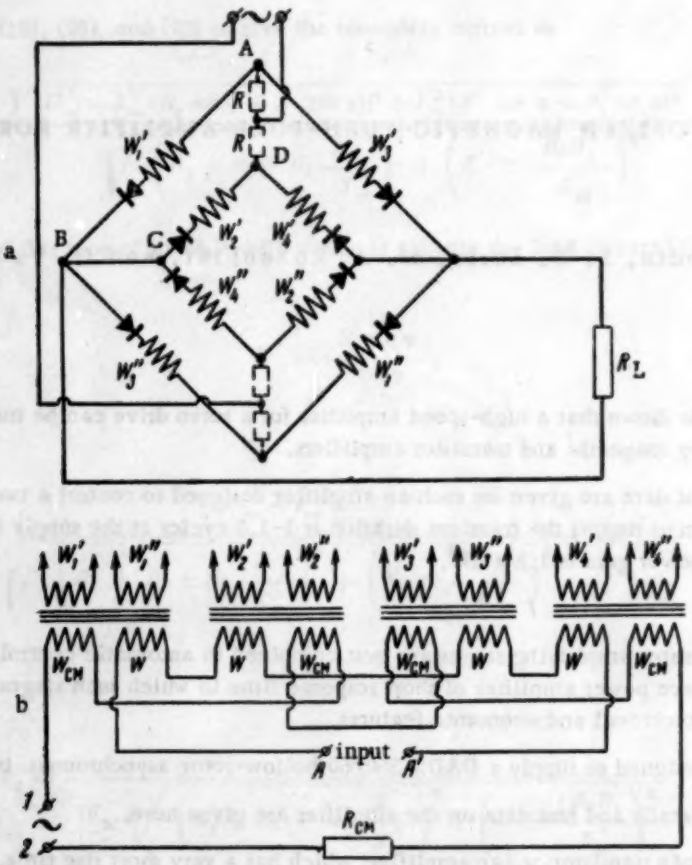

Fig. 1. a) Theoretical diagram of power circuits in the magnetic amplifier, b) control and bias circuits.

One germanium diode type VG-10-55 was connected in series with each W' or W" coil. The maximum long-term output power allowed by the permissible temperature rise exceeded 1 kw.

Figure 2 shows the response with a 35-ohm resistive load. A control current I of 200 ma gave an output power of 1 kw at an efficiency of 0.89, with cos $\varphi = 0.97$. At I = 500 ma the output power was 1.12 kw, with cos $\varphi = 1$ and an efficiency of 0.9.

TABLE 1

Winding	No of turns	Type of wire	Resistance at 20° C, ohms
W'	360	PÉV-1.5	0.8
W"	360	PÉV-1.5	0.91
W	200	PÉV-0.59	3.5
W_{CM}	100	PÉV-0.59	1.8

The maximum power delivered to a 24-ohm load was about 1.5 kw.

Output Stage of the Transistorized Amplifier

The first stages followed normal ac transistor practice, and present no point of interest. We consider only the phase-sensitive stage, which supplies the magnetic amplifier.

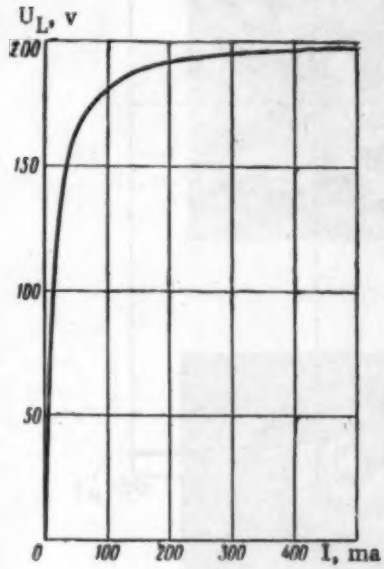

Fig. 2

We first examined the usual circuits for controlling magnetic amplifiers with unrectified ac signals [1-3]. The circuits were found to be more cumbrous than ones in which dc control is used, and, which is more to the point, they gave very much lower power outputs and gains than did the circuit of Fig. 1 (which uses dc control).

Magnetic amplifiers controlled by ac signals are fast, because the transients last no more than 0.5-1.0 cycles of the supply frequency if rectangular-loop cores are used, and if the transient is not extended by a feedback circuit (see [1], p. 664). The same response can, however, also be had from the circuit of Fig. 1, if the earlier amplifying stages are transistorized.

There are many phase-sensitive amplifying circuits with transistors [4, 5]. The circuit of Fig. 3 was used in our amplifier, because it has a high efficiency, a good linearity and a very stable zero.

The voltage U at the input to the phase-sensitive stage (Fig. 3) is in or out of phase with U_C, the voltage from the transformer T_1. The rectified output current I, which controls the magnetic stage (terminals A-A' in Figs. 1b and 3 are common), is determined by U. The sense of I is reversed when U changes 180° in phase.

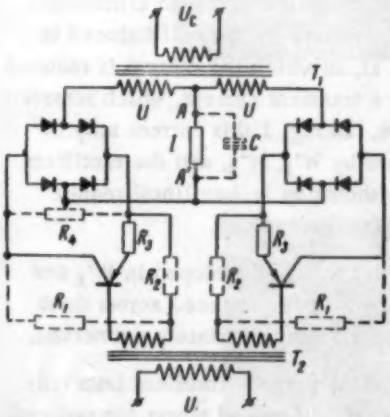

Fig. 3. The transistorized phase-sensitive amplifier.

Type P3 transistors are adequate for the circuit of Fig. 3. The control winding has then to have 400 turns. This value of W causes some lag in the control circuit (see below). Type P4 transistors should be used if this lag is undesirable, and the overload capacity is to be increased over an extended temperature range. These transistors can work less heavily loaded, with W = 200 turns.

Selenium rectifiers type TVS were used in the circuit of Fig. 3.

This circuit has the fault of having a narrow insensitive zone. This zone can be eliminated by applying a small bias to the transistor via R_1 and R_2 (shown by broken lines in Fig. 3). The variable resistor R_4 is used to set the zero.

Figure 4 shows how the load current I in Fig. 3 depends on the signal voltage U when the control windings on the magnetic amplifier are connected. The power gain is about 70 if a P4 transistor is used. A step-signal at the input causes large overvoltages (which may damage the transistor) to appear in the emitter-collector circuit. These overvoltages are removed by shunting the load with the capacitance C (broken lines in Fig. 3). Figure 5 shows how the overvoltage pulse depends on the shunt capacitance when W has 200 turns. The transients last longer if the capacitance is large. The minimum value sufficient to prevent breakdown is used.

Amplifier Characteristics

Figure 6 shows how the output voltage from the magnetic amplifier depends on the input to the first stage when the load resistance is 35 ohms. The over-all power gain is 1.5×10^6 when the load power is 730 watts.

Figure 7 shows oscillograms of transients when W has 400 turns and a P3 transistor is used; a) is for switching-on, and b) for switching-off. The lower curve is the input voltage U, the middle one is the control current I in the magnetic amplifier, and the top one is the output voltage.

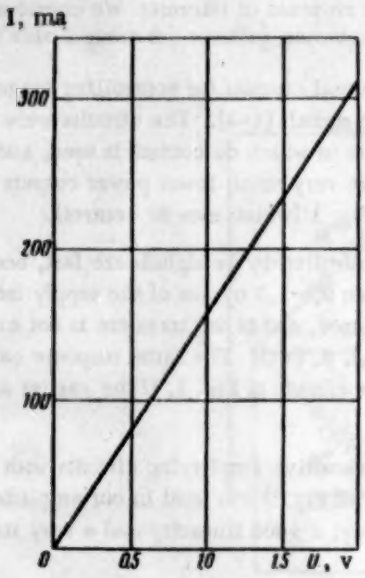

Fig. 4

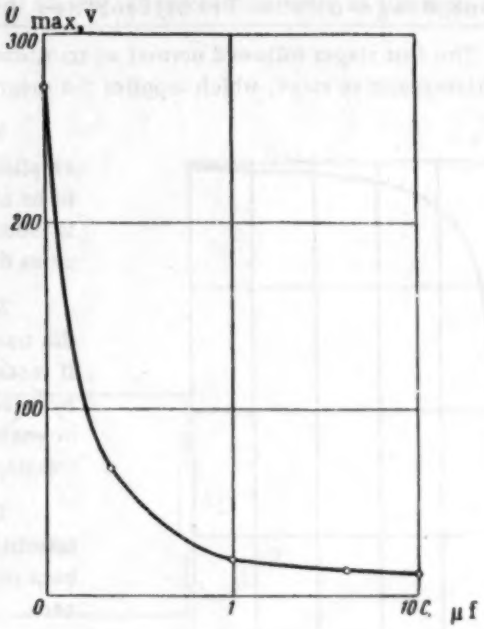

Fig. 5

Figure 8 shows similar curves for W = 200 turns and a P4 transistor. The transient duration is roughly halved, and amounts to 4-5 cycles (0.08-0.1 sec).

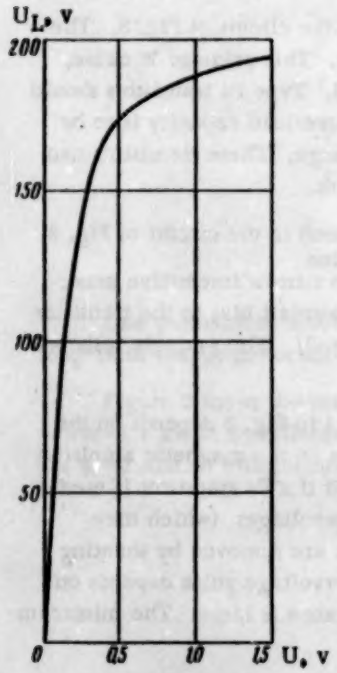

Fig. 6. Response of the complete amplifier working into a resistive load of 35 ohms.

The transient lasts no less time when W has less turns, as Fig. 9 shows; the lag in the control circuit of the magnetic amplifier has been eliminated. The residual lag in the magnetic amplifier is caused by the emf induced in coils W' and W" (e.g., W'_1 and W'_2 in Fig. 1), in which the current is reduced when a signal is received. This emf causes a transient current, which retards the change in the magnetic state of the core. In Fig. 1 this current may be shunted off, e.g., via the loop ABCD formed by W'_1, W'_2, and the rectifiers B'_1 and B'_2. The resistances R (1-1.5 ohms; shown by broken lines) reduce the lag caused by the shunting effect of the feedback loop.

A rectified voltage, of sense opposed to the emf developed in W'_1 and W'_2, W'_3 and W'_4, W''_1 and W''_2 and W''_3 and W''_4, is produced across these resistances during the transient, if the rectifiers are appropriately connected.

Figure 10 relates to R≈1.4 ohms; it is clear that the transient lasts only about two cycles (0.04 sec). The maximum efficiency and power are reduced by 5-8%; by how much, depends on the load resistance. The resistances R can be made much more effective, without reducing the power and efficiency, by smoothing the rectified voltages applied to them. An electrolytic capacitor may, for example, be connected across AD in Fig. 1 to effect this smoothing.

Drive Characteristics

The amplifier was designed to work with a DAD-350/50 two-phase asynchronous motor of the following nominal parameters: output power 372 watts, starting torque 0.4 kg·m, field voltage 127 v, field current 2.6 amp, control voltage 150 v, control current 4.2 amp, speed 2960 rpm, frequency 50 cps.

Figure 11 shows the block diagram. The drive is supplied from the 220-127 v three-phase line (with neutral). The amplifier is connected to wires A and B, and the field winding FW to wires C and O. The voltages on the windings are about 90° apart without a capacitor. The amplifier is designed for continuous running.

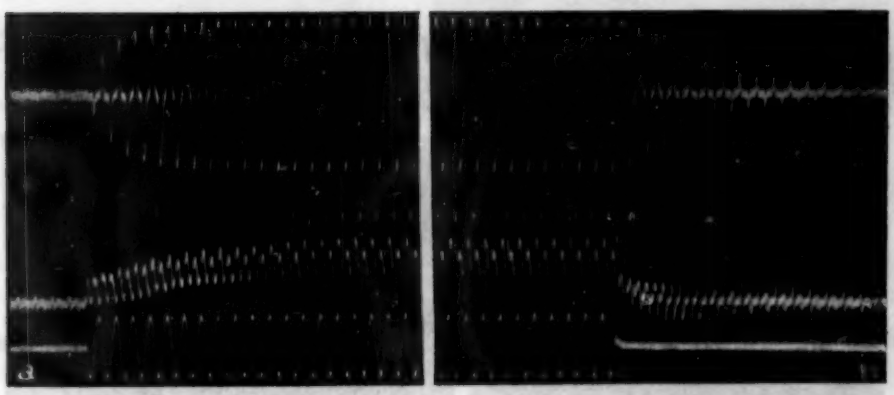

Fig. 7

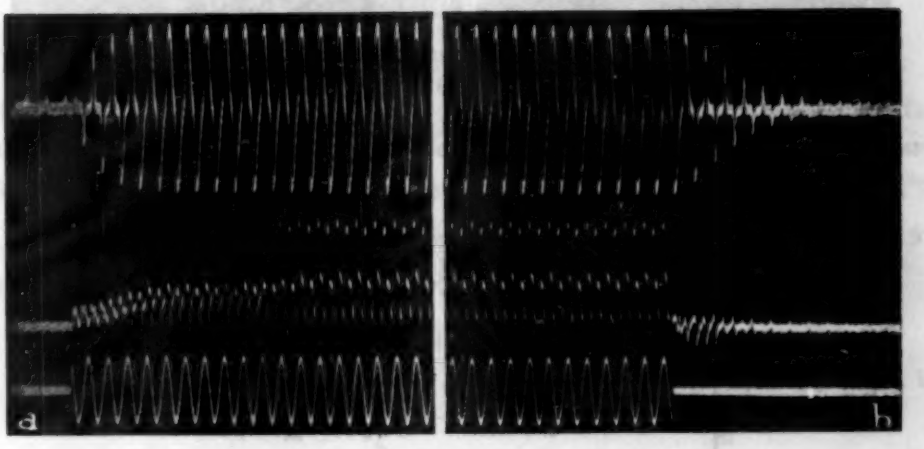

Fig. 8

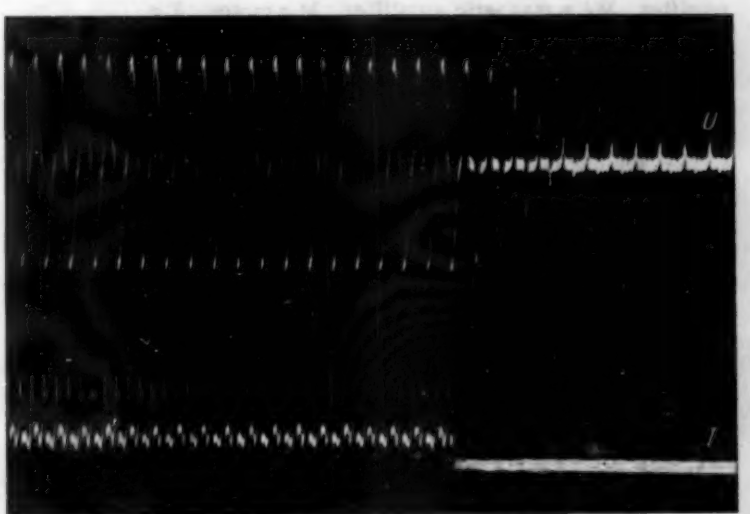

Fig. 9. Oscillograms of the transients in the power circuits of the magnetic amplifier on switching-off, when there is no lag in the control circuit.

We found that the maximum starting torque of 0.38 kg·m was obtained with a control current of 5.6 amp. The maximum control voltage applied to the motor was 180 v, which ensured rapid follow-up.

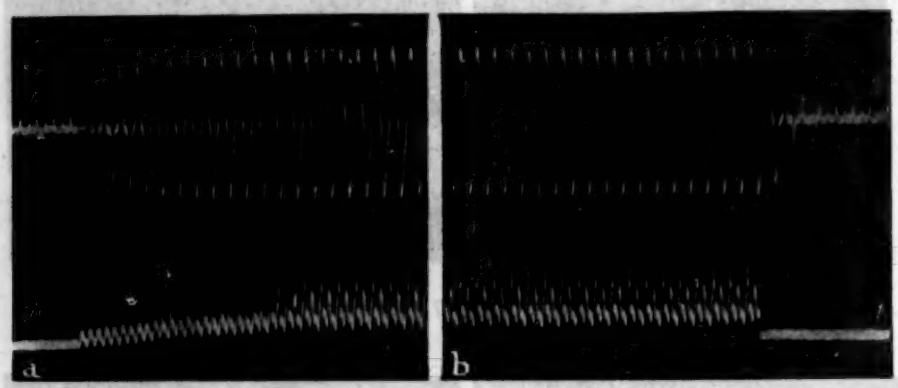

Fig. 10. Oscillograms illustrating the reduced lag in the power circuits obtained by using R = 1.4 ohms. a) Switch-on, b) switch-off.

An ac tachometer was introduced in a feedback loop to ensure that the speed was linearly related to the input signal.

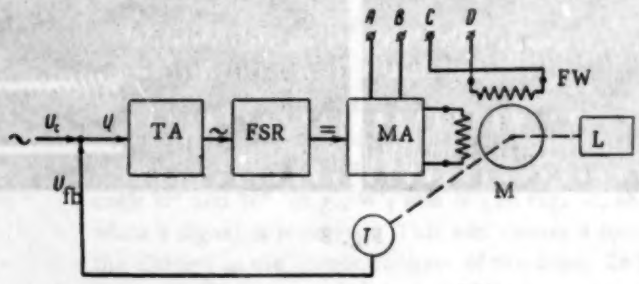

Fig. 11. TA = transistor amplifier, FSR = phase-sensitive rectifier, MA = magnetic amplifier, M = motor, T = = tachometer, L = load.

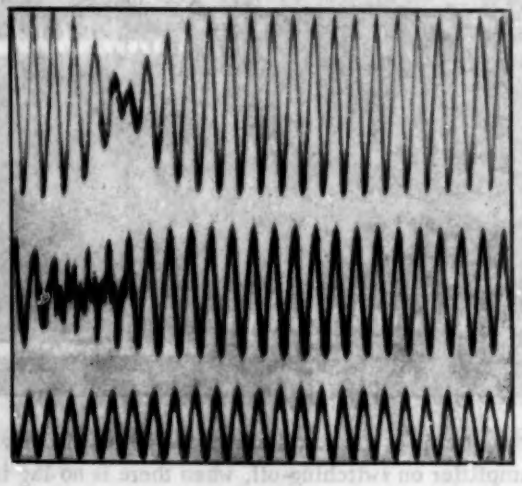

Fig. 12

Figure 12 shows oscillograms for a complete reversal (+2900 to − 2900 rpm) with the loop closed. The lower curve shows the voltage at the input to the first stage, the middle one the output voltage from the magnetic amplifier, and the top one the output from the tachometer (i.e., the motor's speed). The time required for complete reversal was 180-200 msec.

SUMMARY

1. It has been shown that the double-bridge full-wave magnetic amplifier can be used to produce power amplification stages that contain no transformers.

2. Transistorized preamplifier stages can be used to ensure high gain and to reduce the transient duration to 1-1.5 cycles of the supply frequency.

3. The amplifier can give times for complete reversal of 180-200 msec.

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A NEW METHOD OF CONSTRUCTING STABILITY REGIONS FOR AN AUTOMATIC CONTROL SYSTEM IN THE SPACE OCCUPIED BY THE PERMISSIBLE VALUES OF THE PARAMETERS

V. I. Zubov

(Leningrad)

A new method is described for constructing stability regions for an automatic control system in the space occupied by the permissible values of the parameters, in which a characteristic polynomial does not have to be constructed; a way is also given for deriving the parameters that provide a preset transient response.

Problems of the following type are often encountered in automatic control theory [1].

The constant part of a system is given, leaving some choice in the parameters that define the work of the control units. These parameters have to be chosen such that the steady-state motion is stable.

A new method is described for constructing stability regions for an automatic control system in the space occupied by the permissible values of the parameters, in which a characteristic polynomial does not have to be constructed.

1. Deriving the Criterion

Let the steady motion of a control system be specified by the condition $x_1 = x_2 = \ldots = x_n = 0$, where x_1, x_2, x_3, . . . , x_n are coordinates that describe the working. The steady-state motion can then be studied in terms of the solution to a system of equations:

$$\frac{dx_i}{dt} = f_i(t, x_1, \ldots, x_n, \lambda_1, \ldots, \lambda_m) \quad (i = 1, 2, \ldots, n). \tag{1}$$

Here the f_i are real functions, given for $t \geq 0$, $-\infty < x_i < +\infty$ $(i = 1, 2, \ldots, n)$, and Ω is the region occupied by the permissible values of the parameters.

The above condition implies that $f_i(t, 0, \ldots, 0, \lambda_1, \ldots, \lambda_m) \equiv 0$.

Let the set of linear equations

$$\frac{dx}{dt} = \sum_{j=1}^{n} P_{ij} x_j \quad (i = 1, 2, \ldots, n) \tag{2}$$

be a linear approximation to (1). Then the asymptotic stability of the zeroth solution to (2) will [2] imply that the steady motion described by (1) will be stable, while allowing reasonable latitude in respect of the nonlinear terms.

We assume that the coefficients in (2) do not depend on t, but are functions of the parameters.

The zeroth solution to (2) will [3] be asymptotically stable if, and only if, the real parts to the roots of

$$|P-\lambda E|=0 \tag{3}$$

are negative; here P is the matrix for the coefficients in (2), and E is a unit matrix.

In other words, the zeroth solution to (2) will be asymptotically stable if, and only if, all the roots of (3) lie in the left-half of the plane of the complex variable λ.

We image this half-plane on unit circle in the plane of the complex variable ρ with the function

$$\lambda=\frac{\rho-1}{1+\rho}. \tag{4}$$

The roots of (3) then become the roots of

$$\left|P-E\,\frac{\rho-1}{1+\rho}\right|=0. \tag{5}$$

Multiplying (5) by $(1+\rho)^n$, we get

$$|P+E-\rho(E-P)|=0. \tag{6}$$

Replacing ρ by $-\rho$, and multiplying through by $|(P-E)^{-1}|$, we get

$$|B-\rho E|=0,$$

where

$$B=(P-E)^{-1}(P+E). \tag{7}$$

Hence, the zeroth solution to (2) will be asymptotically stable if, and only if, all the roots of (7) lie within a circle of unit radius.

Let these roots be $\rho_1,\ldots,\rho_n$.

The matrix B^k will [4] have roots $\rho_1^k,\ldots,\rho_n^k$. Hence, the zeroth solution to (2) will be asymptotically stable if, and only if, the elements of the matrix B^k tend to zero when $k\to+\infty$. This will be so if, for any k, the modulus of each element in B^k does not exceed α/n, where α is any positive number less than unity, and n is the number of equations.

We thus make the following supposition.

The zeroth solution to (2) will be asymptotically stable if, and only if, there is an integer $k>0$ such that the element b_{ij}^k of the matrix B^k satisfies

$$|b_{ij}^k|\leqslant\frac{\alpha}{n}. \tag{8}$$

2. Deriving the Region of Asymptotic Stability in the Space $\lambda_1,\ldots,\lambda_n$

The above necessary and sufficient criterion can be used for deriving the region of asymptotic stability in the space $\lambda_1,\ldots,\lambda_n$, approximately.

Now (8) was derived by constructing the matrix

$$B=(P-E)^{-1}(P+E)=(P-E)^{-1}[(P-E)+2E].$$

Hence,

$$B=E+2(P-E)^{-1}. \tag{9}$$

Consequently, B can be constructed if we can find the matrix

$$(P - E)^{-1}. \tag{10}$$

Now (10) can be derived from (2) by solving the latter for $x_1, \ldots, x_n$, and then replacing the P_{ii} by $P_{ii} - 1$. Alternatively, instead of (2), we consider

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} (P_{ij} - \delta_{ij}) x_j, \tag{11}$$

where

$$\delta = \begin{cases} 1 & (i = j) \\ 0 & (i \neq j). \end{cases}$$

We find from (11) the $x_1, \ldots, x_n$, which are expressed in terms of

$$\frac{dx_1}{dt}, \ldots, \frac{dx_n}{dt}.$$

The matrix for the coefficients in these expressions will be the same as (10).

Denote the coefficients of (10) by b_{ij}. The zeroth solution to (2) will be asymptotically stable if, and only if, we have

$$|\delta_{ij} + 2b_{ij}| \leqslant \frac{\alpha}{n} < \frac{1}{n} \quad (i, j = 1, 2, \ldots, n). \tag{12}$$

Then (12) defines the region Ω_0 in Ω in which (2) is asymptotically stable.

Let us find the matrix B^2, whose elements are $\sum_{j=1}^{n} b_{ij} b_{jk}$. Then

$$|\delta_{ij} + 2b_{ij} + 4 \sum_{k=1} b_{ik} b_{kj}| \leqslant \frac{\alpha}{n} < \frac{1}{n} \tag{13}$$

defines the region Ω_1 in Ω_0, in which (2) is asymptotically stable.

Hence, (12) and (13) give the stability conditions, which imply that all the roots of (7) lie near the center of a circle $|\rho| < 1$, i.e., that all the roots of (3) lie near $\lambda = -1$, and do not approach the imaginary axis.

Some improvements in the method could be proposed.

3. Some Remarks on the Method

Some notable features of the method can with advantage be emphasized.

1. It enables us to locate a region in which the degree of stability has a pre-set value. Let α denote this pre-set stability, i.e., suppose that the parameters have to be made such that the roots of $|P - \lambda E| = 0$ lie in the half-plane $\mathrm{Re}\,\lambda < \alpha$. Consider the matrix $\overline{P} = P - \alpha E$. The degree of stability will equal α if, and only if, all the eigenvalues of $\overline{P}$ lie in the left half-plane; which condition can be tested by examining the matrix $\overline{B} = E + 2[P - (\alpha + 1)E]^{-1}$.

It is possible to vary α when the stability region is derived on a computer.

2. We choose a circle γ_r in the ρ plane of radius $r < 1$ with its center at the origin. We map the circle $|\rho| < 1$ on the left half-plane of the complex variable λ. Then we get

$$\lambda = \frac{\rho - 1}{\rho + 1}.$$

Hence, it follows that γ_r in the ρ plane becomes a circle c_R in the left half-plane of the complex variable λ. The center of the region c_R lies at the point $\lambda = \frac{r^2+1}{r_2-1}$ on the real axis, and the radius is given by

$$R = \frac{2r}{1-r^2}.$$

Then, if all the eigenvalues of P are to lie within c_R, it is necessary and sufficient that all the eigenvalues of P/r lie within a unit circle. We can then delineate stability regions in the parameter space, within which we have pre-set bounds to the oscillations in the transients.

It is of value to observe that, if we take a number $c < -1$ as center of c_R, the radius is given by

$$R = \frac{1}{2}\sqrt{c^2-1},$$

and r is given by

$$r = \sqrt{\frac{c+1}{c-1}}.$$

3. The general solution to a linear system may [5] be put in the form

$$X = e^{Pt}X_0,$$

where X is a vector with coordinates $x_1, \ldots, x_n$, and $X = X_0$ at $t = 0$.

We assume that α is the largest real part to the roots of P. Then, every coordinate of X can be put in the form

$$x_s = \sum_{i=1}^{n} y_{si}(t)x_i^0 e^{\alpha t} \qquad (s = 1, \ldots, n),$$

where the $y_{si}(t)$ are the elements of the matrix $e^{(P-\alpha E)t}$.

These relations give us that

$$|x_s| \leq \sqrt{\sum_{i=1}^{n} x_i^{02}}\; A_s e^{\alpha t} \qquad (s = 1, \ldots, n),$$

where

$$A_s = \sup_{t>0} \sqrt{\sum_{i=1}^{n} y_{si}^2(t)} \qquad (s = 1, \ldots, n).$$

These latter inequalities are rigorous if $A_1, \ldots, A_n$ are chosen as above.

The amplitudes $A_1, \ldots, A_n$ specify the response. We put $A = \max A_1, \ldots, A_n$ and $q = \frac{\ln A}{|\alpha|}$. Then q clearly specifies the time the transients take to die away more fully than α does. Hence, q may be used as an index to the response; the smaller q, the better the response.

We assume that the canonical structure of P is purely diagonal; then $\overline{P} = S^{-1}M_1S$, where M_1 is a purely diagonal matrix. Now

$$e^{\overline{P}t} = S^{-1}e^{M_1 t}S.$$

Let M_2 denote a diagonal matrix derived from M_1 by transforming the left half-plane of λ into unit circle in the ρ plane.

Then we have

$$\bar{B} = S^{-1} M_2 S,$$

whence

$$\bar{B}^N = S^{-1} M_2^N S = S^{-1} e^{N M_1} S,$$

where $M_1 = \ln M_2$.

These relations imply that matrices $\bar{B}^N$ and $e^{\bar{P}t}$ are closely related. We calculate the number

$$\bar{A} = \sup_{i,k=1,\ldots,n} \sqrt{\sum_{j=1}^{n} b_{ij}^{(k)*}},$$

where the $b_{ij}^{(k)}$ are the elements of the matrix $[\bar{B}]^{2k}$ $(k = 0, 1, \ldots, 7)$.

We put $\bar{q} = \frac{\bar{A}}{|\alpha|}$. Then $\bar{q}$ can be taken as a complete specification of the response.

It is best to delineate subregions, in which $\bar{q}$ does not exceed pre-set limits, when the stability region has been found.

4. The stability in a six-parameter space was calculated on a 'Strela' computer in designing a special system by the above method. The stability region could be found by examining only the traces of several powers of the matrix B (e.g., B^{32}, B^{64}, B^{128}, ...).

The result is of general importance, and can be used to reduce the computing time needed.

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[3] A. M. Lyapunov, The General Problem of Stability of Motion [In Russian].

[4] A. I. Mal'tsev, Fundamentals of Linear Algebra [In Russian] (GTTI, 1956).

[5] N. M. Matreev, Methods of Integrating Systems of Differential Equations [In Russian] (Izd. Leningrad, Gos. Univ., 1956).

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CHOICE OF CHARACTERISTIC FOR NONLINEAR-RATE FEEDBACK IN POSITION SERVOS

B. N. Naumov

(Moscow)

A method for choosing the characteristic for nonlinear-rate feedback in a position servo is given; this feedback is to provide a transient very similar to the desired one with a pre-set structure for the system.

It is shown that the characteristic can be chosen from the experimental response curve for the initially unstabilized servo.

Various nonlinear units have come into use for stabilizing systems and for improving transient responses.

The nonlinear units are chosen to have characteristics such that the transient response is optimal, i.e., takes the minimum time to die away. The devices can be much simpler if we can accept responses that are nearly but not quite optimal.

A method for choosing the characteristic for nonlinear-rate feedback in a position servo is given for the case in which the servo is described by differential equations of any order. The characteristic is chosen from the desired transient response. An example of a system described by a third-order equation is given at the end.

The transient response can be improved by using a nonlinear-rate feedback only for a certain class of inputs, and only for the class of position servos considered here.

This defect occurs, however, with all nonlinear methods that are at present used to improve transient responses.

The Problem

Consider the position servo whose structural diagram is shown in Fig. 1, where $F_1(t)$ is the control input and $x(t)$ is the output. Let $K(p)$ be the transfer function for the fixed part of the open-loop servo, which function contains the transfer functions for the phase-sensitive amplifier FA, for the dc amplifier A, for the amplidyne D, for the motor M, for the reducing gear R, and for the tachometer T.

The transient in an unstabilized high-gain servo is usually very oscillatory, or unstable, in the open-loop state (Fig. 2). The stabilizer is chosen to make the transient stable at the output and have certain given parameters, while the high gain is retained.

A nonlinear-rate feedback, from output to input for instance, can be used to stabilize the system.

This nonlinear-rate feedback need not be taken from the output. It is sometimes better to take it from some internal point. The point chosen naturally depends on the detailed design of the system.

If the follow-up to a change in the input in the initially unstabilized system is at first very rapid, but the transient is later very oscillatory, or unstable, we naturally make the feedback such that it does not affect the start of the transient, but affects only the region in which the steady value $x = x_{ss}$ is approached. We may thus derive the output transient we desire to produce. This transient we may term the sought one.

Figure 3 shows the transient x(t) in the initial system, and the sought one, $x_s(t)$. The feedback has no effect up to $t = t_0$ ($x = x_0$), and so x(t) and $x_s(t)$ coincide. The feedback is in action from $t = t_0$ onwards, and makes the actual transient nearly the sought one.

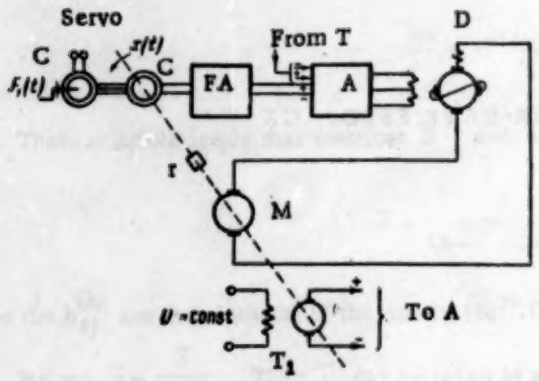

Fig. 1

This method of finding the sought transient is unsound, in that the shape of the transient is chosen arbitrarily. The resulting nonlinear feedback may be complex and difficult to insert.

It is then best to approximate the proper nonlinear response by a simple one, e.g., by a piecewise-linear curve, and to construct the actual transient response, to see if it agrees well with $x_s(t)$.

We transform the structural diagram (with the nonlinear feedback inserted) to the form shown in Fig. 4, where

$$K_l(p) = \frac{K(p)\,p}{1 + K(p)}$$

is the transfer function for the linear part of the open-loop system (we assume that all the parameters in $K_l(p)$ are known),

$$F(t) = L^{-1}\left\{L\{F_1(t)\}\,\frac{K(p)}{1 + K(p)}\right\}$$

is the control input, referred to the input of the nonlinear unit N; this input can be defined in advance, because $F_1(t)$ and K(p) are known;* $x_s(t)$ is as before, and $y = \Phi(x_s)$ is the nonlinear characteristic, which we have to determine.

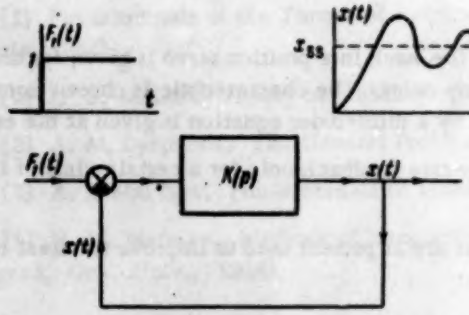

Fig. 2

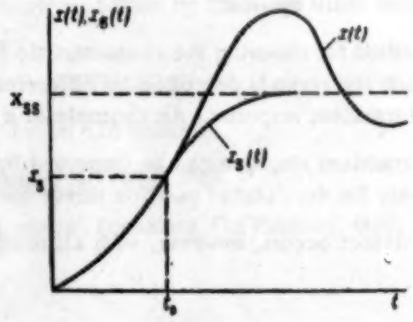

Fig. 3

The problem can then be put in the following form, when we have transformed the diagram to that of Fig. 4.

$K_l(p)$, F(t), and $x_s(t)$ are known; $y = \Phi(x_s)$ must be found.

The Main Relations for the Calculations

We now derive the relations needed in the calculations.

A relation has been given [2-4] which enables us to calculate the transients in nonlinear automatic control systems that have structural diagrams of the type of Fig. 4.

* $F(t) \equiv x(t)$, where x(t) is the output transient from the original servo. This statement is easily proved. Hence, F(t) is usually known, because the initial transient must be calculated before the required transient response can be specified.

This relation is

$$x_{s\,n} = F_n = \left[\frac{\Gamma_0}{2}\Phi_n + \Gamma_1\Phi_{n-1} + \ldots + \Gamma_{n-1}\Phi_1 + \frac{\Gamma_n}{2}\Phi_0\right] \quad (n = 1, 2, 3, \ldots), \tag{1}$$

where

$$x_{s\,n} = x_s(nT), \quad F_n = F(nT), \quad \Gamma_n = Th'_n,$$
$$h'_n = h'(nT), \quad h'(t) = \frac{dh}{dt} = L^{-1}\{K_l(p)\}$$

is the derivative of the response from the linear part, $\Phi_n = \Phi(x_{sn})$, and T is a set time interval.

We should choose T such that the polygon through the x_{sn}, h'_n $(n = 1, 2, 3, \ldots)$ lies reasonably well on the continuous $x_s(t)$ and $h'(t)$ curves. We can derive from (1) a relation that gives us approximate values for the Φ_n $(n = 1, 2, 3, \ldots)$ at the points $x_{s1}, x_{s2}, x_{s3}, \ldots, x_{sn}$.

This relation is

$$\Phi_n = \frac{F_n - x_{s\,n} - \left[\Gamma_1\Phi_{n-1} + \ldots + \Gamma_{n-1}\Phi_1 + \frac{\Gamma n}{2}\Phi_0\right]}{\frac{\Gamma_0}{2}} \quad (n = 1, 2, 3, \ldots) \tag{2}$$

TABLE 1

Typical Table for Calculating the Φ_n

Value of Φ_n	Value of $\frac{\Gamma_0}{2}\Phi_n$	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5
$\frac{\Phi_0}{2}$		$\Gamma_1\frac{\Phi_0}{2}$	$\Gamma_2\frac{\Phi_0}{2}$	$\Gamma_3\frac{\Phi_0}{2}$	$\Gamma_4\frac{\Phi_0}{2}$	$\Gamma_5\frac{\Phi_0}{2}$
Φ_1	$\frac{\Gamma_0}{2}\Phi_1$	$F_1 - x_{s\,1} - \sum_1 = \frac{\Gamma_0}{2}\Phi_1$	$\Gamma_1\Phi_1$	$\Gamma_2\Phi_1$	$\Gamma_3\Phi_1$	$\Gamma_4\Phi_1$
Φ_2	$\frac{\Gamma_0}{2}\Phi_2$		$F_2 - x_{s\,2} - \sum_2 = \frac{\Gamma_0}{2}\Phi_2$	$\Gamma_1\Phi_2$	$\Gamma_2\Phi_2$	$\Gamma_3\Phi_2$
Φ_3	$\frac{\Gamma_0}{2}\Phi_3$			$F_3 - x_{s\,3} - \sum_3 = \frac{\Gamma_0}{2}\Phi_3$	$\Gamma_1\Phi_3$	$\Gamma_2\Phi_3$
Φ_4	$\frac{\Gamma_0}{2}\Phi_4$				$F_4 - x_{s\,4} - \sum_4 = \frac{\Gamma_0}{2}\Phi_4$	$\Gamma_1\Phi_4$
Φ_5	$\frac{\Gamma_0}{2}\Phi_5$					$F_5 - x_{s\,5} - \sum_5 = \frac{\Gamma_0}{2}\Phi_5$

where $\sum_n = \Gamma_1\Phi_{n-1} + \Gamma_2\Phi_{n-2} + \ldots + \Gamma_{n-1}\Phi_1 + \frac{\Gamma_n}{2}\Phi_0 (n = 1, 2, 3, \ldots)$.

The values of Φ_n (n = 1, 2, 3, . . .), are calculated, and the curve for the nonlinear element can then be constructed, because the x_{sn} (n = 1, 2, 3, . . .) are known.

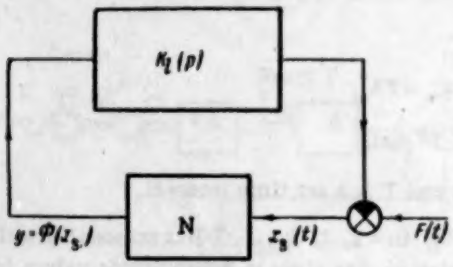

Fig. 4

If $y = \Phi(x_s)$ is complicated and difficult to produce, a simpler shape, which approximates to the best one, should be used, and the actual transient response drawn out. The problem can be taken as solved if the real and best transients differ but little.

The $\Gamma_n = h'_n T = h'(nT)T$ (n = 0, 1, 2, 3, . . .) can be derived only if h'(t) is known at discrete points.

One of the methods given in [5, 6] may be used if this latter curve is known from experiment.

One of the methods used to construct transients for linear systems may be used to find h'(t), because

$$h'(t) = L^{-1}\left\{\frac{K(p)}{1+K(p)}p\right\}.$$

If the sought transient, the input (referred to the nonlinear element) and the transfer function for the linear part of the system are known, it might appear to be enough to substitute $x_s(t)$ and F(t) into the differential equation and thus to determine $y = \Phi(x_s)$. One would then have to differentiate $x_s(t)$ and F(t) repeatedly, however; these functions would be available in graphical or tabular form only, and so the operations would be laborious, and would introduce much more error than the method given here does.

Our method of finding $y = \Phi(x_s)$ is, we consider, very simple and accurate.

It is convenient to use a standardized table (Table 1) in calculating $\Phi_n = \Phi(x_{sn})$ (n = 1, 2, 3, . . .); this Table is like the ones that have been used in calculating transients in nonlinear control systems [1-4].

An Example of Selecting a Nonlinear-Rate Feedback

Consider the system shown in Fig. 1 as an example of selecting a nonlinear rate feedback to give a preset transient.

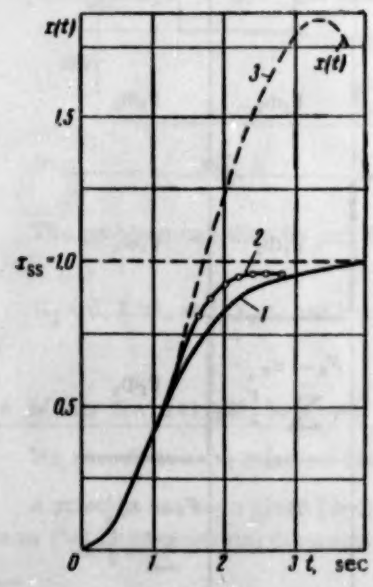

Fig. 5

We assume that the transient is that in response to a step change in $F_1(t)$. We also assume that the initial follow-up rate is low when the tachometer T_1 is absent.

High rates of follow-up to $F_1(t)$ imply that the natural damping must be reduced, or the gain increased. The damping can be reduced by applying positive-rate feedback, for example. In Fig. 1, T_1 effects such feedback. The transient response then becomes very oscillatory, if the initial follow-up rate is high (Fig. 5).

We now assume that the elements in this system are described by linear differential equations.

The corresponding transfer functions are then, respectively:

for the phase-sensitive amplifier FA

$$K_{FA}(p) = k_{FA};$$

for the dc amplifier A

$$K_A(p) = k_A;$$

This relation is

$$x_{s\,n} = F_n = \left[\frac{\Gamma_0}{2}\Phi_n + \Gamma_1\Phi_{n-1} + \ldots + \Gamma_{n-1}\Phi_1 + \frac{\Gamma_n}{2}\Phi_0\right] \quad (n = 1, 2, 3, \ldots), \tag{1}$$

where

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Φ_5	$\frac{\Gamma_0}{2}\Phi_5$					$F_5 - x_{s\,5} - \sum_5 = \frac{\Gamma_0}{2}\Phi_5$

where $\sum_n = \Gamma_1\Phi_{n-1} + \Gamma_2\Phi_{n-2} + \ldots + \Gamma_{n-1}\Phi_1 + \frac{\Gamma_n}{2}\Phi_0 (n = 1, 2, 3, \ldots)$.

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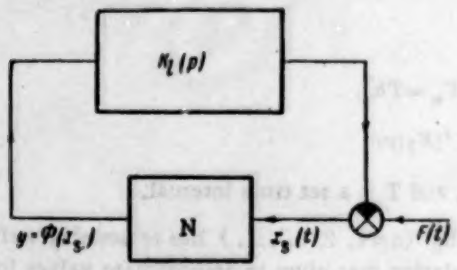

Fig. 4

If $y = \Phi(x_s)$ is complicated and difficult to produce, a simpler shape, which approximates to the best one, should be used, and the actual transient response drawn out. The problem can be taken as solved if the real and best transients differ but little.

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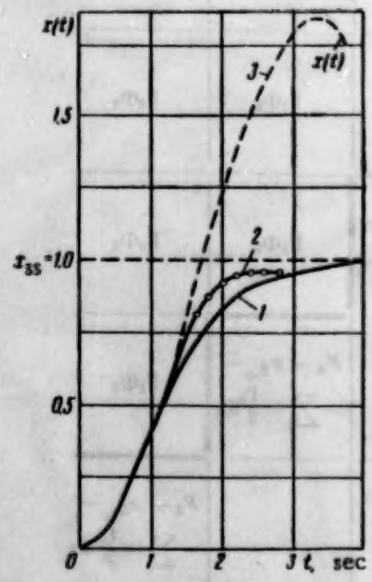


Fig. 5

We assume that the transient is that in response to a step change in $F_1(t)$. We also assume that the initial follow-up rate is low when the tachometer T_1 is absent.

High rates of follow-up to $F_1(t)$ imply that the natural damping must be reduced, or the gain increased. The damping can be reduced by applying positive-rate feedback, for example. In Fig. 1, T_1 effects such feedback. The transient response then becomes very oscillatory, if the initial follow-up rate is high (Fig. 5).

We now assume that the elements in this system are described by linear differential equations.

The corresponding transfer functions are then, respectively:

for the phase-sensitive amplifier FA

$$K_{FA}(p) = k_{FA};$$

for the dc amplifier A

$$K_A(p) = k_A;$$

for the amplidyne D

$$K_D(p) = \frac{k_D}{T_1 p + 1}$$

for the motor M

$$K_M(p) = \frac{k_M}{T_M p^2 + p}$$

for the tachometer T_1

$$K_{T_1}(p) = k_{T_1} p$$

and for the reducing gear R

$$K_r(p) = k_r$$

The transfer function for the initial open-loop system becomes

$$K(p) = \frac{k}{p(a_0 p^2 + a_1 p + a_2)} \tag{3}$$

when the positive rate feedback is applied; here $k = k_{FA} k_A k_D k_M k_r$ is the open-loop gain, $a_0 = T_1 T_M$, $a_1 = T_1 + T_M$, $a_2 = 1 - k_{T_1}$, k_{T_1} is as above, T_1 is the amplidyne's time-constant, and T_M is the dc motor's electromechanical time-constant.*

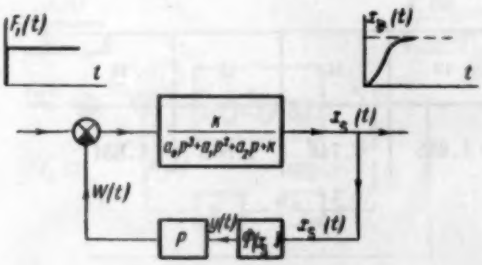


Fig. 6

If x(t), the transient response, is known, it is easy to design the optimum transient (Fig. 5).

We denote the sought (optimum) transient response by $x_s(t)$, and the nonlinear characteristic by $y = \Phi(x_s)$. Figures 6 to 8 show the structural diagram of the system transformed in succession to the diagram of Fig. 4.

Here x(t) (initial transient) will be the same as the control input referred to the input of the nonlinear unit in the modified system, i.e., $F(t) \equiv x(t)$.

The above recurrent relation for $\Phi_n = \Phi(x_{sn})$, ($n = 1, 2, 3, \ldots$) then gives us

$$\Phi_{n-1} = \frac{F_n - x_{s\,n} - [\Gamma_{n-1}\Phi_1 + \Gamma_{n-2}\Phi_2 + \ldots + \Gamma_2 \Phi_{n-2}]}{\Gamma_1} \qquad (n = 1, 2, 3, \ldots), \tag{4}$$

where

$$\Gamma_n = T h'_n, \qquad h'_n = h'(nT), \qquad h'(t) \equiv \frac{d^2 F(t)}{dt^2},$$

and $x_{sn} = x_s(nT)$, $F_n = F(nT)$, and F(t) is the input to the nonlinear unit. We then put

$$z_n = F_n - x_{s\,n} \qquad (n = 1, 2, 3, \ldots). \tag{5}$$

Then (4) becomes

$$\Phi_{n-1} = \frac{z_n - [\Gamma_{n-1}\Phi_1 + \Gamma_{n-2}\Phi_2 + \cdots + \Gamma_2 \Phi_{n-2}]}{\Gamma_1} \qquad (n = 1, 2, 3, \ldots). \tag{6}$$

* We take the parameters as being $a_0 = 0.2$, $a_1 = 1$, $a_2 = 0.3$, and $k = 1$, in future.

We must determine the $z_n = F_n - x_{sn}$ and the Γ_n before we can calculate the Φ_n. Tables 2 and 3 give the F_n and Γ_n calculated for T = 0.2 sec.

The Φ_n are then calculated from a special table (Table 4).

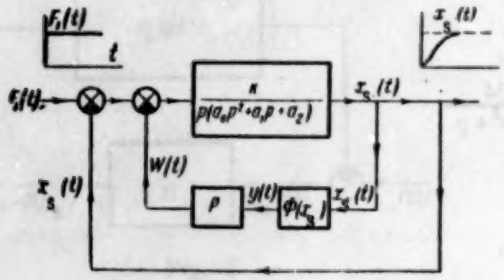


Fig. 7

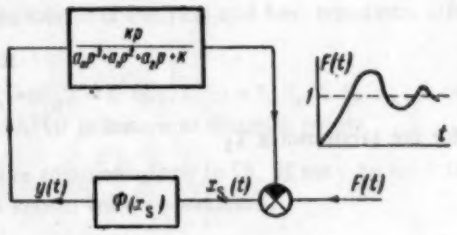


Fig. 8

TABLE 2

Values of F_n (n = 0, 1, 2, 3, . . .), Referred to the Input of the Nonlinear Element for T = 0.2 sec.

n	0	1	2	3	4	5	6	7	8
F(t)	0	0.007	0.050	0.126	0.234	0.368	0.524	0.670	0.876

TABLE 2 (Continued)

n	9	10	11	12	13	14	15	16
F(t)	1.056	1.231	1.393	1.536	1.655	1.746	1.805	1.831

TABLE 3

Values of $\Gamma_n = Th'_n$ (T = 0.2 sec)

n	0	1	2	3	4	5	6	7	8
Γ_n	0	0.124	0.162	0.163	0.148	0.125	0.093	0.059	0.022

TABLE 3 (Continued)

n	9	10	11	12	13	14	15	16	17
Γ_n	0.014	0.050	0.080	—0.110	—0.134	—0.152	—0.163	—0.167	—0.167

Figure 9 shows the $y = \Phi(x_s)$ found in this way. The curve is rather complex. The cause is, of course, that the transient has been chosen arbitrarily. It would be very difficult to reproduce such a curve.

We approximate $\Phi(x_s)$ by a straight line for $|x_s| \geq |x_{s0}|$. We denote the result by $\bar{\Phi}(x_s)$ (Fig. 9). We then calculate the transient response when $\bar{\Phi}(x_s)$ is used. The calculation is not difficult, because Γ_n, $\bar{\Phi}_n$, and F_n are known.

Figure 5 shows the transient response when $\bar{\Phi}(x_s)$ is used. The differences from $x_s(t)$ are small; the approximate transient is easy to realize, because $\bar{\Phi}(x_s)$ is simple to produce.

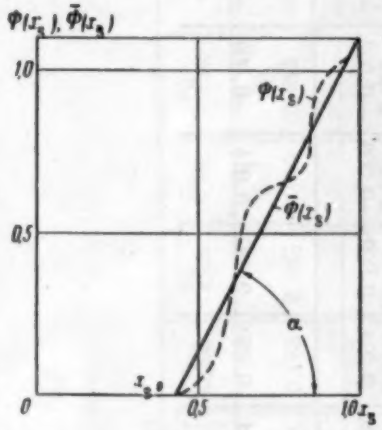


Fig. 9

A diode circuit [7] can be used to produce the nonlinear response. The nonlinear element must, however, incorporate an amplifier, because the slope of $\bar{\Phi}(x_s)$ is greater than unity.

Now the nonlinear response must vary with the size of the step in $F_1(t)$, if the transient response is to remain nearly optimal; the insensitive range in $\bar{\Phi}(x_s)$ must vary in proportion to $|F_1(t)|$. If a diode circuit produces $\bar{\Phi}(x_s)$, the cathode voltage has to vary with $F_1(t)$. Figure 10 shows the structural diagram of the system with this type of nonlinear element. The nonlinear response can also be produced in other ways.

The nonlinear feedback for a given $F_1(t)$ is effected by the following computational operation:

$$W(t) = \frac{d\bar{\Phi}(x_s)}{dt} = \frac{d\bar{\Phi}(x)}{dx_s}\frac{dx_s}{dt}. \qquad (7)$$

Now $\frac{d\bar{\Phi}(x)}{dx_s}$ can be reproduced with a relay having an insensitive zone (Fig. 11), where $\left|\frac{d\bar{\Phi}(x_s)}{dx}\right| = |\tan\alpha|$, where $\tan\alpha$ is the slope of the inclined part in $\bar{\Phi}(x_s)$.

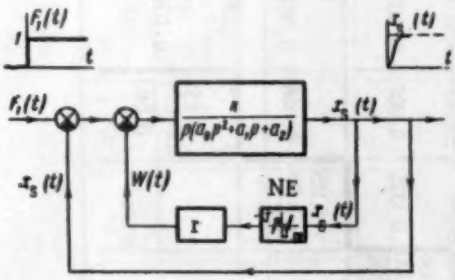


Fig. 10

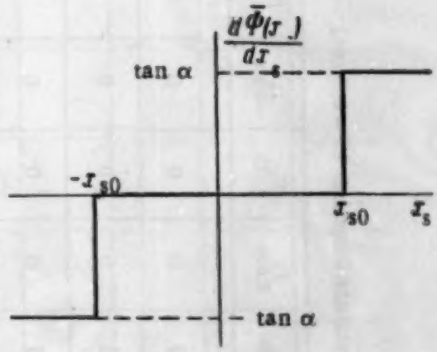


Fig. 11

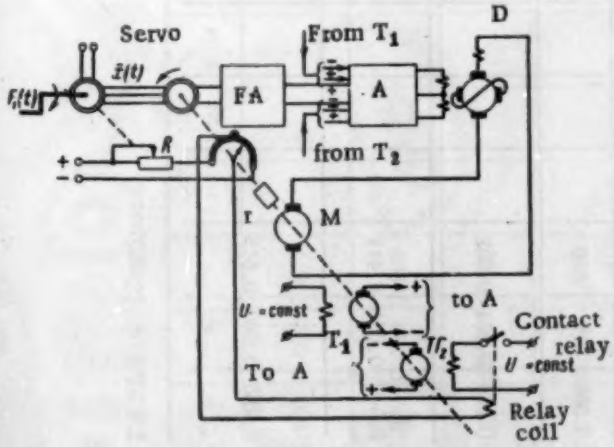


Fig. 12

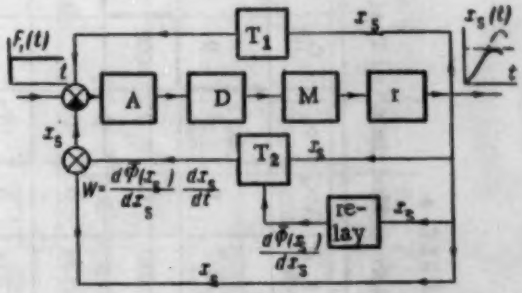


Fig. 13

TABLE 4

Table to Find the $\Phi(x_s)$ That Provides the Sought Transient Response ($T = 0.2$ sec)

x_s n	$\Gamma_1\Phi_n$	Φ_n	Γ_2 0.162	Γ_3 0.163	Γ_4 0.148	Γ_5 0.125	Γ_6 0.093	Γ_7 0.059	Γ_8 0.022	Γ_9 −0.014	Γ_{10} −0.050	Γ_{11} −0.060	Γ_{12} −0.110	Γ_{13} −0.134	Γ_{14} −0.152	Γ_{15} −0.163	Γ_{16} −0.167	Γ_{17} −0.167
0.007	0	$\Phi_0/2$ 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.050	0	0	0.000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.126	0	0		0.000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.234	0	0			0.000	0	0	0	0	0	0	0	0	0	0	0	0	0
0.368	0	0				0.000	0	0	0	0	0	0	0	0	0	0	0	0
0.580	0.004	0.032					0.004 −0.000 0.004	0.005	0.005	0.005	0.004	0.003	0.002	0.001	0	0	0	0
0.640	0.051	0.411						0.056 −0.005 0.051	0.066	0.067	0.061	0.051	0.038	0.024	0.009	−0.006	−0.002	−0.003
0.786	0.080	0.645							0.151 −0.071 0.080	0.104	0.105	0.095	0.081	0.060	0.030	0.014	−0.009	−0.003

TABLE 4 (Continued)

$x_3 n$	$\Gamma_1\Phi_n$	Φ_n	Γ_2 0.162	Γ_3 0.163	Γ_4 0.148	Γ_5 0.125	Γ_6 0.093	Γ_7 0.059	Γ_8 0.022	Γ_9 −0.014	Γ_{10} −0.050	Γ_{11} −0.060	Γ_{12} −0.110	Γ_{13} −0.134	Γ_{14} −0.152	Γ_{15} −0.163	Γ_{16} −0.167	Γ_{17} −0.167
0.800	0.084	0.677								0.260 −0.176 0.084	0.110	0.110	0.100	0.085	0.063	0.040	0.015	−0.009
0.850	0.100	0.806									0.380 −0.280 0.100	0.130	0.131	0.110	0.101	0.075	0.047	0.018
0.885	0.119	0.960										0.508 −0.389 0.119	0.155	0.156	0.142	0.120	0.089	0.057
0.915	0.114	0.919											0.621 −0.507 0.114	0.149	0.150	0.136	0.115	0.085
0.935	0.126	1.016												0.720 −0.594 0.126	0.164	0.166	0.150	0.127
0.950	0.129	1.040													0.796 −0.667 0.129	0.168	0.169	0.154
0.973	0.119	0.960														0.832 −0.713 0.119	0.155	0.156
0.980	0.132	1.064															0.851 −0.719 0.132	0.172
0.986	0.084	0.677																0.838 −0.754 0.084

Here we have to use a multiplier to produce $W(t)$, e.g., a tachometer (T_2) with independent excitation, and a relay that switches on this excitation when $x_s(t) = x_{s0}$. Here T_1, as before, is to reduce the natural damping (i.e., to increase the initial follow-up rate).

Figure 12 shows a system with this type of nonlinear-rate feedback, and Fig. 13 the corresponding structural diagram.

The insensitive zone can be coupled to changes in $F_1(t)$ by the resistance R (Fig. 12).

Figures 14 and 15 show the transients found on an electronic analog. Figure 14 relates to the linear system with positive-rate feedback, and Fig. 15 to the system with the nonlinear feedback.

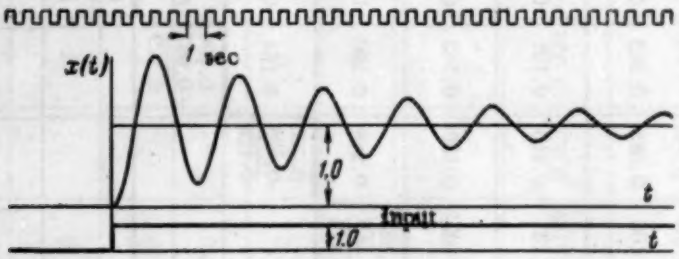
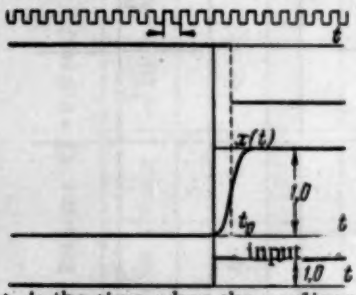


Fig. 14

Figure 16 shows the transient obtained when linear stabilizing elements are used.

It is clear from Figs. 15 and 16 that the nonlinear feedback gives a very much more brief transient.



t_0 is the time when the nonlinear feedback is switched-in.

Fig. 15

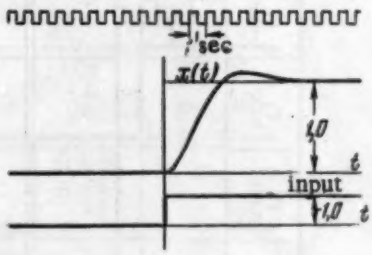


Fig. 16

SUMMARY

The method enables one to find a nonlinear-rate feedback that will, for a given structure in the position servo, provide a transient response near to optimal in form.

The method imposes no restriction on the order of the differential equation that describes the fixed part of the system.

The experimental transient response of the initial (unstabilized) system can be used in selecting the feedback characteristic.

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METHODS OF STUDYING STABILITY IN AUTOMATIC CONTROL SYSTEMS WITH DISTRIBUTED PARAMETERS

Ya. B. Kadymov

(Baku)

Several papers [1-6] deal with methods of studying stability in automatic control systems with distributed parameters.

Most of these papers deal with classes of system for which the characteristic equations can be put in the form

$$\psi_1(p)\,e^{p\tau} + \psi_2(p)\,e^{-p\tau} = 0. \tag{1}$$

Others [3, 6] deal with ones whose characteristic equations can be put in the form

$$\psi_1(p)e^{p\tau} + \psi_2(p)e^{-p\tau} + \psi_3(p) = 0 \tag{2}$$

In some practical problems, e.g., in rotary drilling, in which longitudinal and transverse oscillations occur, the characteristic equations can be put in the form

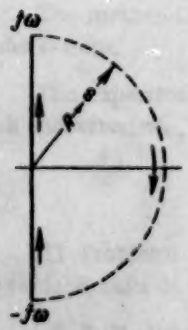
$$\psi_1(p)\,[a_1e^{p\tau_1} + a_2e^{-p\tau_1}]\,[a_3e^{p\tau_2} + a_4e^{-p\tau_2}] - k\psi_2(p) = 0, \tag{3}$$

where $\psi_1(p)$ and $\psi_2(p)$ are polynomials of degrees $\underline{m}$ and $\underline{c}$, with $m \geq c$; τ_1, τ_2, a_1, a_2, a_3, a_4, and $\underline{k}$ are real parameters, with $\tau_2 = n\tau_1$ ($\underline{n}$ is any positive integer).

Consider the function

$$\varphi_\tau(p) = \frac{\psi_1(p)\,[a_1e^{p\tau_1} + a_2e^{-p\tau_1}]\,[a_3e^{p\tau_2} + a_4e^{-p\tau_2}] - k\psi_2(p)}{\psi_1(p)\,a_1a_3e^{p(\tau_1+\tau_2)}} =$$

$$= \left(1 + \frac{a_2}{a_1}\,e^{-2p\tau_1}\right)\left(1 + \frac{a_4}{a_3}\,e^{-2p\tau_2}\right) - \frac{k\psi_2(p)}{\psi_1(p)\,a_1a_3}\,e^{-p(\tau_1+\tau_2)}. \tag{4}$$



The zeros of this function equal the roots of (3), while the poles equal the roots of $\psi_1(p) = 0$. We apply to $\varphi_\tau(p)$ the argument principles, and get

$$\frac{1}{2j\pi}\oint\frac{\varphi_\tau'(p)}{\varphi_\tau(p)}\,dp = s - r$$

or

$$\Delta \arg \varphi_\tau(p) = 2\pi\,(s - r), \tag{5}$$

where $\underline{s}$ is the number of zeros with positive real parts, and $\underline{r}$ is the number of poles with positive real parts.

Consider a closed curve in the complex plane P which consists of the imaginary axis and a half-circle of radius R (see figure).

The function $\varphi_T(p)$ tends to unity as we move along the half-circle as R tends to infinity; hence the integral, and the change in the argument of $\varphi_T(p)$, will tend to zero.

The integral differs from zero only for movements along the imaginary axis.

Hence, we get the changes in the argument to $\varphi_T(p)$ by putting $p = j\omega$ in (5), where ω varies from $-\infty$ to $+\infty$:

$$\Delta \arg \varphi_T(j\omega) \Big|_{\infty}^{-\infty} = 2\pi(s - r). \tag{6}$$

If the poles of $\varphi_T(p)$ lie on the imaginary axis, then we may replace the regions in which such poles appear by half-circles of infinitely small radius; we then apply the argument principles to get an expression analogous to (6).

The system will be stable if there are no zeros in the right half of the plane ($r = 0$), i.e., if

$$\Delta \arg \varphi_T(j\omega) \Big|_{\infty}^{-\infty} = -2\pi r.$$

If the system with $\psi_1(p) = 0$ as its characteristic equation is stable, we have $r = 0$, and so the change in the argument is

$$\Delta \arg \varphi_T(j\omega) \Big|_{\infty}^{-\infty} = 0. \tag{7}$$

Then the condition for stability can be formulated as follows: the system will be stable if the total number of turns made by $\varphi_T(j\omega)$ in the positive direction is zero when ω changes from $-\infty$ to $+\infty$.

We substitute into (7) for $\varphi_T(j\omega)$ and get

$$\Delta \arg \left\{ \left(1 + \frac{a_2}{a_1} e^{-2j\omega\tau_1}\right)\left(1 + \frac{a_4}{a_3} e^{-2j\omega\tau_2}\right) - \frac{k\psi_2(j\omega)}{\psi_1(j\omega)\, a_1 a_3} e^{-j\omega(\tau_1+\tau_2)} \right\} \Big|_{\infty}^{-\infty} = 0. \tag{8}$$

The critical values $\tau = \tau_{0i}$ define where $\varphi_T(j\omega)$ passes through the origin, i.e., where $\varphi_T(j\omega)$ has imaginary roots.

The stability boundaries will correspond to small changes in τ_{0i} values that change $\varphi_T(j\omega)$ in such a way as to satisfy the stability condition.

The condition that defines the critical τ_{0i} values is

$$\left[1 + \frac{a_2}{a_1} e^{-2j\omega\tau_1}\right]\left[1 + \frac{a_4}{a_3} e^{-2j\omega\tau_2}\right] - \frac{k\psi_2(j\omega)}{\psi_1(j\omega)\, a_1 a_3} e^{-j\omega(\tau_1+\tau_2)} = 0. \tag{9}$$

It is difficult to solve this equation, because the boundaries must be determined for different τ's. We multiply (9) by $e^{j\omega(\tau_1+\tau_2)}$ and get

$$\left(e^{j\omega\tau_1} + \frac{a_2}{a_1} e^{-j\omega\tau_2}\right)\left(e^{j\omega\tau_2} + \frac{a_4}{a_3} e^{-j\omega\tau_2}\right) - \frac{k\psi_2(j\omega)}{\psi_1(j\omega)\, a_1 a_3} = 0. \tag{10}$$

We put $x = \omega\tau_1$. Then, $nx = \omega\tau_2$, and (10) becomes

$$\left(e^{jx} + \frac{a_2}{a_1} e^{-jx}\right)\left(e^{jnx} + \frac{a_4}{a_3} e^{-jnx}\right) - \frac{k\psi_2(j\omega)}{\psi_1(j\omega)\, a_1 a_3} = 0 \tag{11}$$

or

$$\Omega(jx) = \lambda(j\omega),$$

where

$$\Omega(jx) = \left(e^{jx} + \frac{a_3}{a_1} e^{-jx}\right)\left(e^{jnx} + \frac{a_4}{a_2} e^{-jnx}\right)$$

and

$$\lambda(j\omega) = \frac{k\psi_3(j\omega)}{\psi_1(j\omega)\, a_1 a_2}.$$

We give ω and $\underline{x}$ values from $-\infty$ to $+\infty$, and get $\Omega(jx)$ and $\lambda(j\omega)$ curves. The points where these curves intersect define the solution to (9), i.e., define the critical values τ_{0i} and ω_{0i}.

Following [4], we differentiate stable, unstable and neutral systems. We examine the stability in two stages.

Stage one. We construct $\Omega(jx)$ as a function of $\underline{x}$, and $\lambda(j\omega)$ as a function of ω, and superimpose the curves to get $\underline{i}$ intersections, which correspond to the critical values of ω_{0i} and τ_{0i}. This stage amounts to finding the critical τ_{0i}.

Stage two. We establish whether the limiting system is stable, using the usual tests.

If the system is stable, then [4] the condition $\tau = 0 < \tau_{01} < \tau_{02} < \ldots < \tau_{0n}$ gives us the stability regions $0 - \tau_{01}$, $\tau_{02} - \tau_{03}$, $\tau_{04} - \tau_{05}$, etc.

If the system is unstable or neutral, we determine the regions of stability and instability as in [4]. The only difference is that in [4] we have $\lambda(j\omega)$ intersecting a circle, whereas here we have it intersecting the curve $\Omega(jx)$. If $n = 0$, $\Omega(jx)$ will represent an ellipse. If $x = 0$ and $a_2 = 0$ we get the equations considered in [4].

We consider the characteristic equation for the initial system, and assume that the characteristic equation for the limiting system has $r = 0$. Those equations with $r \neq 0$ are dealt with in a similar way; the assumptions made in [4] are used.

This method is based on separating the variables $x = \omega\tau$ and ω. It can also be used with equations of the type of (2). We divide all the terms in (2) by $\psi_3(p)$ and substitute $p = j\omega$; we get

$$\varphi_1(j\omega)\, e^{j\omega\tau} + \varphi_2(j\omega)\, e^{-j\omega\tau} + 1 = 0,$$

where

$$\varphi_1(j\omega) = \frac{\psi_1(j\omega)}{\psi_3(j\omega)} \quad \text{and} \quad \varphi_2(j\omega) = \frac{\psi_2(j\omega)}{\psi_3(j\omega)}. \tag{12}$$

The conjugate equation is

$$\overset{*}{\varphi}_1(j\omega) e^{-j\omega\tau} + \overset{*}{\varphi}_2(j\omega) e^{j\omega\tau} + 1 = 0. \tag{13}$$

We solve (12) with (13) to get

$$\frac{\overset{*}{\varphi}_1(j\omega) - \varphi_2(j\omega)}{\varphi_1(j\omega) - \overset{*}{\varphi}_2(j\omega)} = e^{2j\omega\tau}$$

or

$$\xi(j\omega) = e^{2j\omega\tau}.$$

The points where $\xi(j\omega)$ cuts a circle of unit radius define the critical values of τ_{0i}.

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SIMULATING NUCLEAR POWER SYSTEMS

B. Ya. Kogan, Yu. A. Nechaev, and F. E. Tranin

(Moscow)

Some ways of using electronic analogs to solve nuclear power problems are indicated. Results are given for some reactor kinetics and poisoning studies made on electric analogs.

Nuclear power systems have recently become very important. The simplest such system (Fig. 1) has a reactor (which produces heat from a chain fission reaction) fitted with a coolant system and with a control rod system, one (or several) coolant circulation loops, and a system that utilizes the heat (turbine, etc.).

The equations to be solved include those for the reactor kinetics, those for the thermodynamics, hydrodynamics and mechanics of the various components, and those for the various external feed-back loops in the automatic control system.

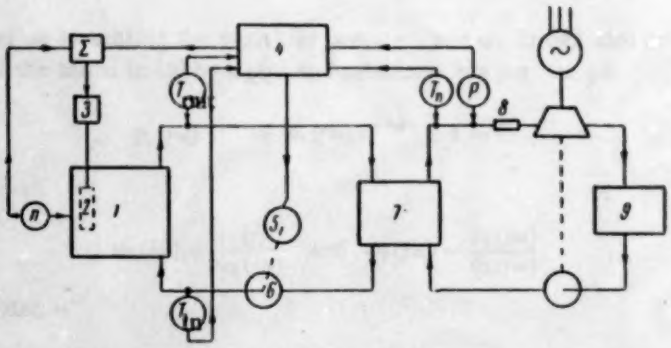

Fig. 1. Skeleton of a nuclear power system. 1) Reactor, 2) control rod, 3) servo, 4) program controller, 5) pump drive, 6) pump, 7) heat exchanger, 8) control valve on turbine, 9) condenser.

The equations cannot be solved analytically, and the basic data are often not very accurate, and so electronic analogs can be used to good advantage.* The reactor's response to changes in the system can be predicted, the automatic control system can be adjusted to give optimum response, the actual control apparatus can be checked by connecting it to a reactor analog, and experiments not free from hazard in the real system can be conducted safely. The concentrations of fission-product poisons can be calculated, and the analogs can be used in training staff for nuclear power stations. Some reactor kinetics and poisoning topics are dealt with here.

* Electronic analogs have also been much used abroad for solving nuclear power problems (see [1-19], for instance).

1. The Basic Reactor Kinetics Equations

We consider only transient states in the reactor proper, and write the equations, assuming that the fuel is fixed, that the effective multiplication coefficient depends linearly on the mean temperature in the core, and that the heat loss follows Newton's law.

a) The neutron balance equation is

$$\frac{dn}{dt} = -\frac{\beta n}{l} + \frac{(1-\beta)\,\delta k_e}{l}\,n + \sum \lambda_i C_i, \tag{1}$$

where n is the total number of neutrons at time t, k_e is the effective multiplication coefficient ($k_e = 1 + \delta$), l is the mean lifetime of a prompt neutron, C_i is the number of fission fragments responsible for the i-th group of delayed neutrons, λ_i is the decay constant of these fragments, and β is the ratio of delayed to total fission neutrons.

b) The equation for the number of fission fragments responsible for the i-th group of delayed neutrons is

$$\frac{dC_i}{dt} = \frac{\delta k_e + 1}{l}\,\beta_i n - \lambda_i C_i. \tag{2}$$

Here β_i is the ratio of the i-th group of delayed neutrons to total fission neutrons.

The number of equations in (2) depends on how many delayed-neutron groups are allowed for. The number does not usually exceed six. The delayed photoneutrons can be allowed for by introducing an extra group.

Often one average group is used instead of the six groups; the parameters of this group are derived as follows:

$$\beta = \sum_i \beta_i \text{ and } \lambda = \frac{\sum_i \beta_i}{\sum_i \frac{\beta_i}{\lambda_i}}.$$

c) The heat balance equation is

$$\frac{d\overline{T}}{dt} = \mu^*(P_v - P_{v\,0}), \tag{3}$$

where $\overline{T}$ is the mean temperature in the core, μ^* is a constant defined by the physical properties of the core materials, P_V is the power produced per unit volume from fission, and P_{VO} is the power extracted from unit volume.

We assume that the power is proportional to the total number of neutrons ($P_V = kn$), and that $P_{VO} = \alpha_V^* (\overline{T} - \overline{T}_C)$ is proportional to the difference between the mean core temperature and the temperature of the incoming coolant (α_V^* is a volume heat loss coefficient); we then have

$$\frac{d\overline{T}}{dt} = \mu\,[n - \alpha_v(\overline{T} - \overline{T}_{\,C})], \tag{4}$$

where $\alpha_v = \frac{\alpha_v^*}{\xi}$, $\mu = k\mu^*$.

d) The equation for the effective multiplication coefficient is

$$\delta k_e = k_e - 1 = \delta k_r - \alpha(\overline{T} - \overline{T}_i) + \delta k_c = \delta k_r - \alpha\Delta\overline{T} + \delta k_c, \tag{5}$$

where δk_r is the part of the effective multiplication coefficient determined by the control system, α is a temperature coefficient, $\overline{T}_i$ is the initial core temperature, $\Delta\overline{T}$ is the deviation from the initial temperature, and δk_c is the random component of the effective multiplication coefficient.

We assume that we are given $\delta k_r(t)$, $\delta k_c(t)$, and $\overline{T}_C(t)$ for the referred system (1) - (5).

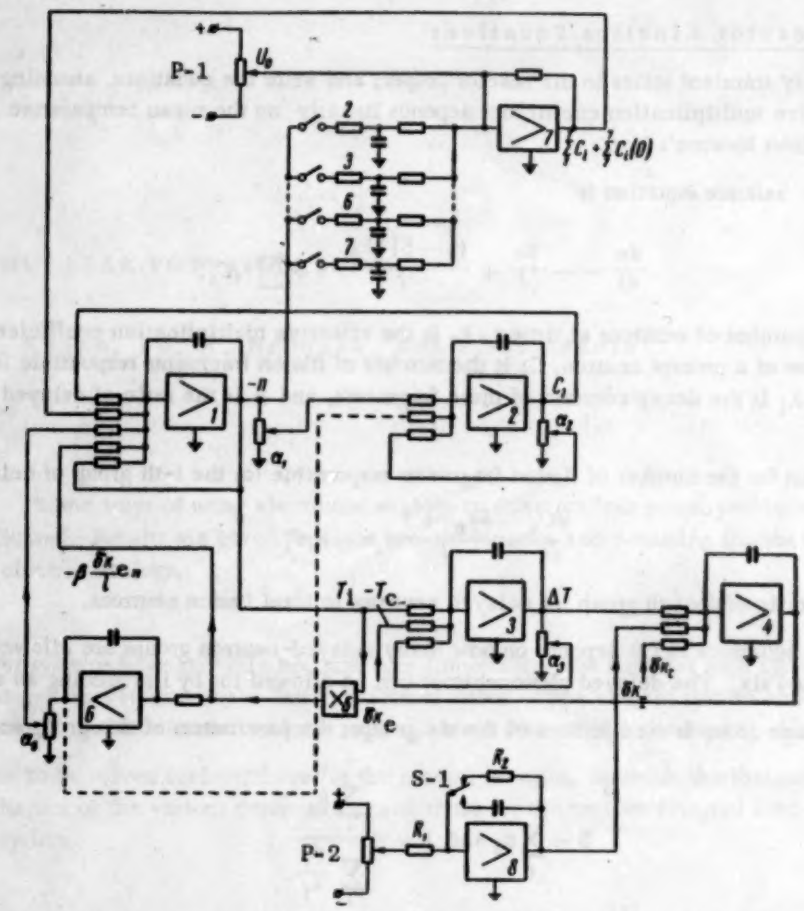

Fig. 2. The analog to a nuclear reactor. α_1, α_2, α_3, and α_4 are voltage division coefficients.

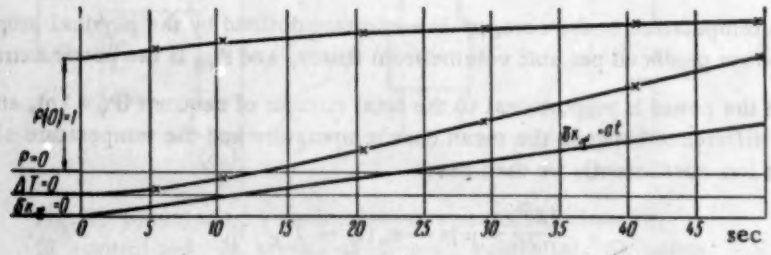

Fig. 3

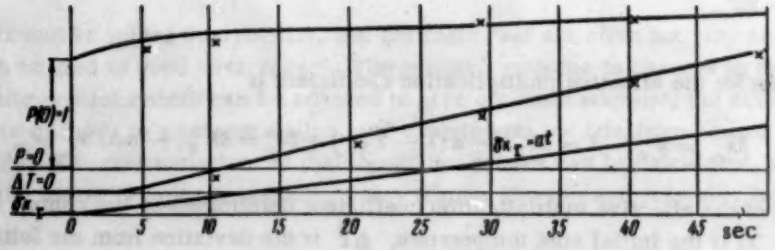

Fig. 4

We also assume that the initial conditions are known from the condition that the previous state was a steady one:

$$\overline{T}(0) = \overline{T}_1 \quad , \; n(0) = \alpha_v [\overline{T}_1 \quad - \overline{T}_c \quad (0)] \text{ and } C_i(0) = \frac{\beta_i}{l\lambda_i} n(0).$$

2. Structural Diagrams of the Analog, and Allowance for the Delayed-Neutron Groups

Figure 2 shows the structural diagram of the analog used to reproduce (1) - (5) by reducing the order of the derivative [20]. Parts of the equations for the delayed-neutron groups are reproduced by passive circuits connected to the input of the summing amplifier 7. The initial conditions for these groups are set by means of the voltage U_0 taken from the potentiometer P-1.

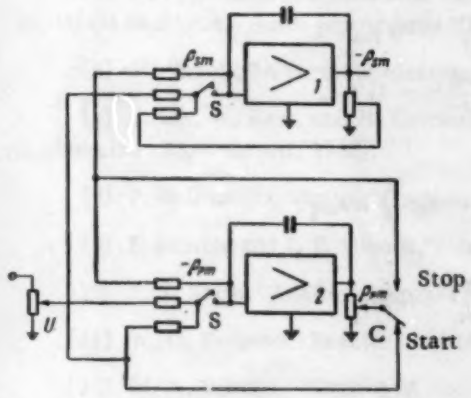

Fig. 5. Circuit to reproduce the changes in samarium concentration during operation and shutdown.

It is best to use operations amplifiers if the separate groups of delayed neutrons are to be reproduced. The link shown by a broken line has been shown to have little effect in practice. The potentiometer P-2, the switch S-1 and the integrator 8 serve to apply a δk_r which varies linearly in time, or stepwise. The curves in Figs. 3 and 4 show the transients in $\Delta\overline{T}$ and P when δk_r follows the law $\delta k_r = at$; the curves relate to one group of delayed neutrons only, and to six groups, respectively. The oscillogram curves of Figs. 3 and 4 are supplemented by points calculated on a digital computer for $\Delta\overline{T}$ and P for one average delayed-neutron group. The difference between the digital and analog results does not exceed 5%. Replacing the six delayed groups by one average one affects the neutron flux result only at the very start.

3. Analogs to Poisoning Processes

The isotopic composition of the fuel varies during the reaction. Some fission products have large neutron-capture cross sections, and so affect (reduce) the multiplication coefficient. The effects of start-stop cycles on the poison concentrations, and thus on the effective multiplication coefficient, must be known if the reactor is to be used correctly.

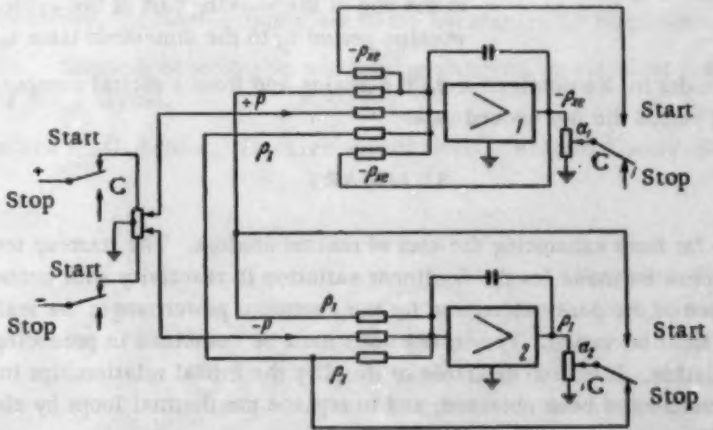

Fig. 6. Circuit to reproduce the changes in xenon concentration during operation and shutdown. α_1 and α_2 are voltage division coefficients.

The transients in P last very much less longer than do the changes in the poison concentrations. We can therefore assume that P changes stepwise from P = 0 to some value P = const ('start'), or from P = const to P = 0 ('stop'). Table 1 gives the equations for the concentrations, in which only the Xe and Sm (and their daughter products I and Pm) are allowed for.

TABLE 1

Equations for Poison Concentrations

Poison	Operating conditions	
	fixed power $P = \text{const}$	shutdown $P = 0$
Sm^{149}	$\frac{d\rho_{Sm}}{dt} + PA\rho_{Sm} = \lambda_1\rho_{Pm}$, $\frac{d\rho_{Pm}}{dt} + \lambda_1\rho_{Pm} = BP$	$\frac{d\rho_{Sm}}{dt} = \lambda_1\rho_{Pm}$, $\frac{d\rho_{Pm}}{dt} = -\lambda_1\rho_{Pm}$
Xe^{135}	$\frac{d\rho_{Xe}}{dt} + (\lambda_2 + CP)\rho_{Xe} = \lambda_3\rho_I + DP$, $\frac{d\rho_I}{dt} + \lambda_3\rho_I = EP$	$\frac{d\rho_{Xe}}{dt} = -\lambda_2\rho_{Xe} + \lambda_3\rho_I$, $\frac{d\rho_I}{dt} = -\lambda_3\rho_I$

The notation is P is the power, the ρ's are the numbers of the atoms of the respective elements per cm^3, and λ_1, λ_2, λ_3, A, B, C, D and E are known constants.

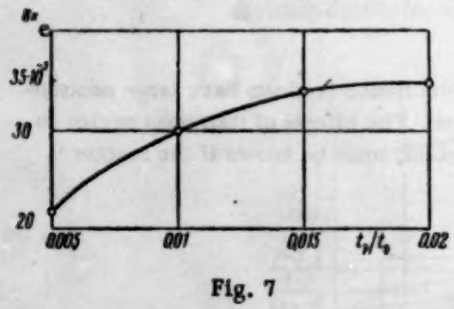

Fig. 7

Figures 5 and 6 show the structural diagrams of the circuits used to set-up these equations. The stop relay S and the operating conditions relay C, both controlled from the time program mechanism, effect the changes from stop to start, etc. Reactors may operate or be shut down for hundreds of hours, and so appropriate time scales (often different for operating and shutdown conditions) are used.

Figure 7 shows how the effective multiplication varies up to the end of the working part of the cycle, as a ratio of the working period t_p to the shut-down time t_0.

We compared the results for Xe obtained from the analog and from a digital computer; the error in ρ_{Xe} at the end of an operating period did not exceed 4%.

SUMMARY

These examples are far from exhausting the uses of reactor analogs. The start-up transients are of great interest. Here allowance must be made for the nonlinear variation in reactivity with control-rod position, for the temperature-dependence of the parameters, and for the enormous power range; the scales in which the quantities are represented must be varied. Very many units must be combined in producing an analog to a complete nuclear power installation. It is then desirable to simplify the initial relationships in some reasonable fashion after the analog results have been obtained, and to replace the thermal loops by electrical equivalents, e.g., by adjustable delay lines.

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THE ANALYTIC THEORY OF CONSTRUCTING ROOT LOCI

G. A. Bendrikov and K. F. Teodorchik

(Moscow)

Graphical and semianalytic ways of finding roots (natural frequencies) have been given [1-8], and are now largely standard. A new, purely analytic, way of constructing trajectories is of some interest, however.

Let the characteristic equation of a linear closed system have the form *

$$f_n(p) = p^n + \bar{a}_{n-1}\, p^{n-1} + \ldots + \bar{a}_1 p + \bar{a}_1 = 0. \tag{1}$$

Let us see how the roots move in the complex plane as the parameter ρ, which appears in the coefficients in (1), varies. ** To find the trajectories in terms of ρ, we substitute into (1) an expression

$$\bar{p} = \delta + i\omega. \tag{2}$$

We separate the real and imaginary terms, and get

$$P(\delta, \omega, \rho) + i\omega Q(\delta, \omega, \rho) \equiv 0. \tag{3}$$

Now (3) shows that, as ρ varies, the roots follow trajectories such that

$$P(\delta, \omega, \rho) = 0 \quad \text{and} \quad \omega Q(\delta, \omega, \rho) = 0. \tag{4}$$

The complete induction method [10] shows that P and Q can be represented as finite series:

$$\begin{aligned} P(\delta, \omega, a_k) &= f_n(\delta) - \frac{\omega^2}{2!} f_n''(\delta) + \frac{\omega^4}{4!} f_n^{IV}(\delta) - \ldots = 0, \\ Q(\delta, \omega, a_k) &= f_n'(\delta) - \frac{\omega^2}{3!} f_n'''(\delta) + \frac{\omega^4}{5!} f_n^{V}(\delta) - \ldots = 0. \end{aligned} \tag{5}$$

The structures of these series are important. The series are polynomials in even powers of ω only, with coefficients that are also polynomials in integral powers of δ. The a_k of (1) enter linearly into these latter polynomials. The a_k also bring in the variable ρ.

The general equations for the root loci of (4) allow of two types of solution.

1. If $f_n'(\delta) \neq 0$, (5) shows us that the solution for $\omega = 0$ gives the real roots of

$$f_n(\delta, \rho) = 0. \tag{6}$$

* Such an equation is expressed as follows in terms of the transfer function for the open-loop system: $1 + W_0(p) = 0$.

** We envisage systems that are stable for some values of ρ. All the fixed coefficients are therefore taken as being positive. The coefficients that depend on ρ are taken as being also positive, at least within the range of change in ρ.

This equation enables us to separate out sections on the real axis along which the real roots of (1) move as ρ varies within certain limits. These sections can be found either directly by varying ρ and finding the appropriate δ from (6), or (which is sometimes simpler) by solving (6) for ρ:

$$\rho = \psi(\delta) \tag{7}$$

and calculating ρ for a set of values of δ.

We consider only loci that relate to stable systems; it is then enough to take δ within the limits $-a_{n-1} \leq \delta \leq 0$.* In both cases we draw up curves in δ, ρ coordinates.

If we have that $f^*(\delta, \rho) = 0$ and (6) both apply, the corresponding point on the real axis is, for that value of ρ, a repeated root.

2. The solution for $\omega \neq 0$ gives

$$P(\delta, \omega, \rho) = 0 \text{ and } Q(\delta, \omega, \rho) = 0. \tag{8}$$

These equations define the loci of the complex roots in the (δ, ω, ρ) space. These loci, projected on the (δ, ω) plane, are just the loci in the complex plane. The latter loci can be constructed directly if (8) can be put in the form

$$R(\delta, \rho) = 0 \text{ and } S(\delta, \omega) = 0. \tag{9}$$

If we give δ values for which (1) remains stable, we can, from (9), construct the root loci in the $(\delta, i\omega)$ plane, and the curves for the variable parameter in the (δ, ρ) plane. Two such sets of curves of course completely define the behavior as ρ varies [10].

We end our note on the bases of this general analytic method by considering cases for which the analysis can be carried further in a general form. We return to (5). The constant term in (1) appears only in the first equation of (5).

This latter fact enables us to consider at once the case in which the variable parameter appears only in a_0 in (1). This case is of real interest, because it relates to changing the gain in a servo that has a transfer function $W_0(p)$ with no zeros.

We note that, if

$$W_0(p) = \frac{k \prod_1^m (p + z_k)}{\prod_1^n (p + p_i)},$$

where k is a variable parameter, the characteristic equation for the closed loop system will have the form

$$\prod_1^n (p + p_i) + k \prod_1^m (p + z_k) = 0. \tag{10}$$

This equation contains the variable gain in linear form in the $(m+1)$ terms of lowest degree. We have the case in which the parameter appears only in the constant, if there are no zeros ($m = 0$). Then the second equation in (5) contains only δ and ω^2, and so, for systems up to those of the sixth order, the equation

$$Q(\delta, \omega^2) = 0 \tag{11}$$

contains ω^2 to powers no higher than the second.

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* The limits may be made ($-1 \leq \delta \leq 0$) by introducing in (1) a new variable (assuming $a_{n-1} = \text{const} > 0$), which is $p = a_{n-1} z$. All the roots are then reduced by a factor a_{n-1}, but their relative positions are unchanged.

Thus, there are only arithmetical difficulties in constructing the root loci for systems up to those of the sixth order. The first equation in (5) is linear in the sole variable coefficient $a_0(\rho)$, and so gives us directly the value of ρ that corresponds to any root $\overline{p} = \delta + i\omega$.

This simple case has been considered in detail elsewhere [9, 10].

We have a much more general case if ρ appears in linear form in several coefficients in (1). Then (1) becomes

$$p^n + a_{n-1}p^{n-1} + \ldots + a_1 p + a_0 + \rho\,(b_m p^m + b_{m-1}p^{m-1} + \ldots + b_1 p + b_0) = 0 \tag{1'}$$

or in abbreviated form

$$\Phi_n(p,\, a_k) + \rho\varphi_m(p,\, b_i) = 0 \tag{12}$$

where $m \leq n$ is the largest subscript to the coefficients containing ρ.

The real-root loci are given directly by (12) in the form

$$-\rho = \frac{\Phi_n(\delta)}{\varphi_m(\delta)}\,. \tag{13}$$

Now (5), for the complex roots, takes the form

$$\begin{aligned} P(\delta,\, \omega,\, \rho) &= \left[\Phi_n(\delta) - \frac{\omega^2}{2!}\Phi_n''(\delta) + \ldots\right] + \rho\left[\varphi_m(\delta) - \frac{\omega^2}{2!}\varphi_m''(\delta) + \ldots\right] = 0, \\ Q(\delta,\, \omega,\, \rho) &= \left[\Phi_n'(\delta) - \frac{\omega^2}{3!}\Phi_n'''(\delta) + \ldots\right] + \rho\left[\varphi_m'(\delta) - \frac{\omega^2}{3!}\varphi_m'''(\delta) + \ldots\right] = 0. \end{aligned} \tag{14}$$

We eliminate ρ, and get the complex root loci as

$$\begin{aligned} &\left[\Phi_n(\delta) - \frac{\omega^2}{2!}\Phi_n''(\delta) + \ldots\right]\left[\varphi_m(\delta) - \frac{\omega^2}{3!}\varphi_m'''(\delta) + \ldots\right] - \\ &-\left[\Phi_n'(\delta) - \frac{\omega^2}{3!}\Phi_n'''(\delta) + \ldots\right]\left[\varphi_m(\delta) - \frac{\omega^2}{2!}\varphi_m''(\delta) + \ldots\right] = 0. \end{aligned} \tag{15}$$

The value of ρ corresponding to a given root is found from

$$-\rho = \frac{\Phi_n(\delta) - \frac{\omega^2}{2!}\Phi_n''(\delta) + \ldots}{\varphi_m(\delta) - \frac{\omega^2}{2!}\varphi_m''(\delta) + \ldots} = \frac{\Phi_n'(\delta) - \frac{\omega^2}{3!}\Phi_n'''(\delta) + \ldots}{\varphi_m'(\delta) - \frac{\omega^2}{3!}\varphi_m'''(\delta) + \ldots}\,. \tag{16}$$

This case is of some interest, because it relates to the effects of the zeros in the transfer function for the open-loop system on the root locus for the closed-loop system, if the gain is the variable parameter (see [11]).

Consider a system with an open-loop transfer function that has four poles and one zero

$$W_0(p) = \frac{k\,(p + z)}{p\,(p + p_1)\,(p + p_2)\,(p + p_3)}\,. \tag{17}$$

The characteristic equation for the closed-loop system is

$$\Phi_4(p,\, a_k) + k\varphi_1(p,\, b_l) = (p^4 + a_3p^3 + a_2p^2 + a_1p) + k\,(p + b_0). \tag{18}$$

We take $\underline{k}$ as the variable parameter and use (15) and (16); we get an equation for the loci of the real roots:

$$\omega = 0 \text{ and } -\overline{k} = \frac{\Phi_4(\delta)}{\varphi_1(\delta)} \tag{19}$$

and an equation for the loci of the complex roots:

$$\frac{1}{4!}\Phi_4^{IV}(\delta)\omega^4 - \left[\frac{1}{2!}\Phi_4''(\delta) - \frac{1}{3!}\Phi_4'''(\delta)\varphi_1(\delta)\right]\omega^2 + [\Phi_4(\delta) - \Phi_4'(\delta)\varphi_1(\delta)] = 0 \qquad (20)$$

and

$$k = \Phi_4'(\delta) - \frac{\omega^2}{3!}\Phi_3'''(\delta). \qquad (21)$$

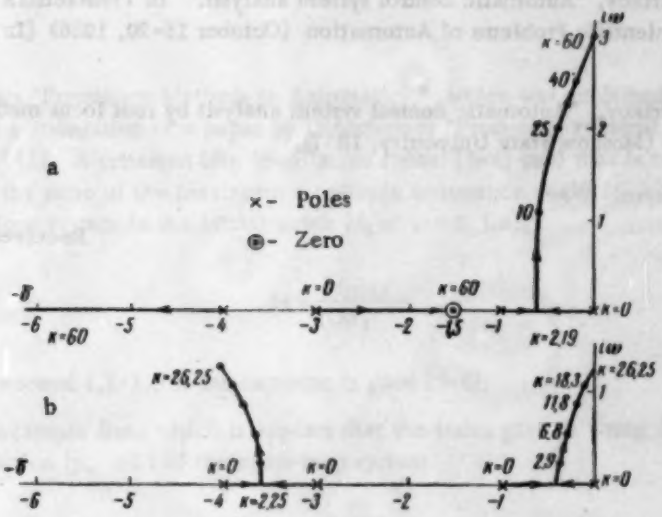

Fig. 1

Figure 1, a shows the root loci in the $(\delta, i\omega)$ plane for

$$W_0(p) = \frac{k(p+1.5)}{p(p+1)(p+3)(p+4)}. \qquad (22)$$

Figure 1, b shows the effects of the zero in $W_0(p)$ for a system in which

$$W_0(p) = \frac{k}{p(p+1)(p+3)(p+4)}. \qquad (23)$$

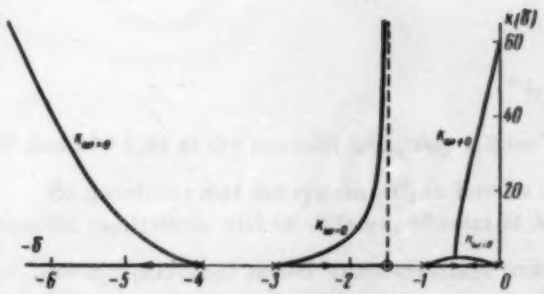

Fig. 2

Figure 2 shows $k = k(\delta)$ for a system with the transfer function of (22). The branch $k_{\omega \neq 0}$ corresponds to values of k that give the complex roots, and the branch $k_{\omega = 0}$ corresponds to values that give the real roots.

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LETTERS TO THE EDITOR

The collection of papers "Frequency Methods in Automatics," which was published by the Foreign Literature Press in 1957, contained a translation of a paper by Oldenburger "Frequency response data presentation, standards and design criteria" [1]. A criterion (the 'oscillation index' [2-4] one) that is often used is given in this paper, and is defined as the ratio of the maximum amplitude (resonance peak) M_{max} on the frequency response curve for the closed-loop system to the initial value M_0 at $\omega = 0$, i.e.,

$$M = \frac{M_{max}}{M_0}$$

This index should not exceed 1.1-1.5 if the damping is good [2-4].

Oldenburger gives an example from which it appears that the index gives a wrong idea of the damping. He considers the transfer function (p. 59) of the open-loop system

$$G = \frac{0.486}{s^2 + 0.3s + 1}$$

and asserts that, if the closed-loop system has simple unit feedback, the roots of the characteristic equation will be

$$\delta_{1,2} = -0.15 \pm j1.21$$

and that $M = 1.34$ at the resonant frequency $1.2\ \text{sec}^{-1}$.

He concludes that the system will in fact be unstable if the real and imaginary parts have this ratio, because the oscillations will be so large, whereas M does not exceed the permitted limits.

An inexperienced reader would conclude from this example that the index is unsuitable as a test for tendencies to oscillate, because it gives wrong results.

We give the correct value for the index for Oldenburger's example. The closed-loop transfer function

$$\Phi = \frac{G}{1+G} = \frac{0.486}{s^2 + 0.3s + 1.486}$$

corresponds to a resonant frequency of $1.2\ \text{sec}^{-1}$.

This leads to $M_{max} = 1.34$.

The initial value (for $\omega = 0$) is

$$M_0 = \frac{0.486}{1.486} = 0.327.$$

Then

$$M = \frac{M_{max}}{M_0} = 4.1.$$

Hence the system will in practice be unsuitable.

Thus, Oldenburger's example in fact shows that the criterion indicates correctly whether the system is liable to oscillate; it does not show that the criterion is unreliable.

The error arises because the height of the resonance peak was taken without reference to the initial amplitude. This neglect is permissible only if this initial value is in fact unity.

The latter situation occurs if, for example, the system is astatic. If it is not, as in Oldenburger's example, the index will be the relative height of the resonance peak for the closed-loop system.

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V. A. Besekerskii

ANGLE AND LINEAR DISPLACEMENT DIGITIZERS

A. D. Talantsev

(Moscow)

Angle and linear displacement digitizers are classified by their manner of quantization and methods of coding. There are two methods of quantizing physical scales: by means of interval indentification or discrete-points identification. Converters with read-off type coding are grouped by their method of eliminating false codes. Typical circuits and designs used in practice are described.

Among the analog-to-digital converters* there is a large group of devices designed to convert spatial quantities (angles of rotation, linear displacements, etc.) into a digital form. Such devices are used in automatic control systems which contain as their elements digital computers (digital systems of lathe control, digital fire control units, pilotless navigation systems, etc.) [1-5].

Another important sphere of application of analog-to-digital converters of this type is the digital recording of parameters in production processes in factories [6 and 7].

Converters are made by many foreign instrument making firms [8 and 9].

An attempt is made in the present review, based on foreign periodic publications, to classify these devices.

This article is part of the investigation of the analog-to-digital converter principles of design conducted by the IARC of the Academy of Sciences, USSR under the guidance of V. A. Trapeznikov.

1. Basic Operations

Analog-to-digital conversion consists of two basic operations: quantizing and coding [10 and 11].

Quantizing. In quantizing, the measured quantity (analog) is compared with a standard, and substituted by another quantity close to it in value and consisting of a whole number of standard units. For this purpose a quantizing scale is produced, which consists of standard stretches equal to, or multiples, of the standard unit. The latter is sometimes called elementary quantum of the scale [12 and 13]. For future reference let us call this quantity q. If the beginning of counting is set on the scale, the scale stretches equal to a whole number of standard units form levels of quantization.

When the quantized scale is reproduced some of the identified points are given physical features distinct from those of the remaining points.

Two methods of building up the scale should be distinguished: the method of identifying intervals between points (Fig. 1a) and the method of identifying discrete points (Fig. 1b).

Fig. 1

For quantizing the spatial scale, physical properties connected with electrical, magnetic, optical, mechanical and other phenomena are used. Such properties can consist of conductivity and resistance, magnetization and the lack of it, transparency and opaqueness, protrusion and cavity, etc.

* They are also called: continuous-discrete converters, digitizers, etc.

The quantized scale should only be considered in conjunction with the device for distinguishing the physical features of the scale. This device must have a source of energy in the form of a generator of physical values (electrical current, magnetic or light flux, etc.) and a reproducing element (b on Fig. 1a and 1b). The latter is in the majority of cases a transducer of selected features into electrical quantities (photocell, pick-up head, etc.).

Converters of spatial quantities into a digital form can have a scale quantized with respect to another physical property, such as a time or electrical scale. In conjunction with this, other standards are chosen such as time intervals, or sources of emf or current, etc. In such cases, the measured angles or linear displacements must be first converted into time or electrical quantities.

Coding. Coding consists in establishing correspondence between each quantization level and a certain aggregate of conventional signs called a code-combination or group. The conventional signs occupy places or positions in the time of spatial scale (positional-method of digital representation). Digital elements of any one type are used for coding.

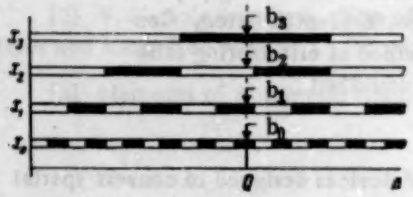

Fig. 2. Coding by means of a field of code combinations.

Methods of coding are distinguished by the type of digital elements and the manner of filling the positions [11, 13]. Spatial quantities are coded either by the method of counting the quanta of the scale or by the method of reading off the code of the quantized scale.

When coding by the counting method the scale quanta are converted into electrical pulses which are counted by electronic counters. The counting of the number of quanta can be made periodically (starting from zero) or additively only, when the measured value changes.

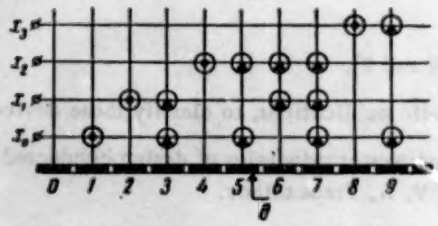

Fig. 3. Matrix method of coding.

For coding by the reading-off method, code combinations for all the scale levels are established in advance, independently of the measured quantity. This can be fulfilled by two methods. The first is illustrated in Fig. 2. A number of quantized scales with identified intervals are placed parallel to axis A in such a manner that the beginning of their counts coincide. The scales have different quanta values. Figure 2 shows the case when $q_0 = q$, $q_1 = 2q$, $q_2 = 4q$, . . . , $q_{n-1} = 2q^{n-1}$. Let us call the aggregate of quantized scales, each of which serves to fill with signs the corresponding position of the code, a field of code combinations.

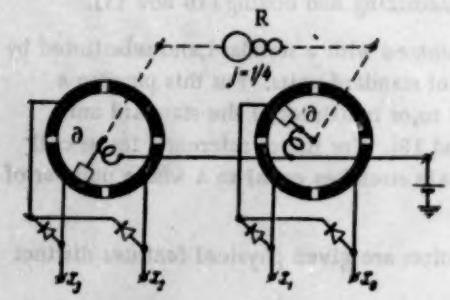

Fig. 4. Reduction of an angle in matrix coding.

Coding by means of code combinations is direct. The physical properties of the code itself serve as conventional signs of the code. When the field is moved with respect to the sensing elements b_0, b_1, b_2, and b_3, the latter "read" the codes of their categories and henceforth their indications are considered as the conventional signs of the code at level Q.

The system of coordinates used for the field of code combinations depends on the character of the measured quantity. If a linear displacement is measured, the field is constructed in Cartesian coordinates, as shown in Fig. 2. When angles are measured, the field is usually constructed in the polar system of coordinates, with the code signs distributed along the radius. Coding by means of reading-off does not normally represent quanta levels in a digital manner. Codes can be assigned to levels in a completely arbitrary manner. Let us assume that the latter is typical of a coding system (or a "code system") of analog-to-digital converters. However, quite independently of any code distribution rule along the quanta levels, correspondence can be established between these levels and a set of digits by means of counting the quanta. Such a system of coding is known as the natural (or "binomial") system [11, 13]. This system can be considered as a "reference system" with respect to which all the other systems are considered

ANGLE AND LINEAR DISPLACEMENT DIGITIZERS

A. D. Talantsev

(Moscow)

Angle and linear displacement digitizers are classified by their manner of quantization and methods of coding. There are two methods of quantizing physical scales: by means of interval indentification or discrete-points identification. Converters with read-off type coding are grouped by their method of eliminating false codes. Typical circuits and designs used in practice are described.

Among the analog-to-digital converters* there is a large group of devices designed to convert spatial quantities (angles of rotation, linear displacements, etc.) into a digital form. Such devices are used in automatic control systems which contain as their elements digital computers (digital systems of lathe control, digital fire control units, pilotless navigation systems, etc.) [1-5].

Another important sphere of application of analog-to-digital converters of this type is the digital recording of parameters in production processes in factories [6 and 7].

Converters are made by many foreign instrument making firms [8 and 9].

An attempt is made in the present review, based on foreign periodic publications, to classify these devices.

This article is part of the investigation of the analog-to-digital converter principles of design, conducted by the IARC of the Academy of Sciences, USSR under the guidance of V. A. Trapeznikov.

1. Basic Operations

Analog-to-digital conversion consists of two basic operations: quantizing and coding [10 and 11].

Quantizing. In quantizing, the measured quantity (analog) is compared with a standard, and substituted by another quantity close to it in value and consisting of a whole number of standard units. For this purpose a quantizing scale is produced, which consists of standard stretches equal to, or multiples, of the standard unit. The latter is sometimes called elementary quantum of the scale [12 and 13]. For future reference let us call this quantity q. If the beginning of counting is set on the scale, the scale stretches equal to a whole number of standard units form levels of quantization.

When the quantized scale is reproduced some of the identified points are given physical features distinct from those of the remaining points.

Two methods of building up the scale should be distinguished: the method of identifying intervals between points (Fig. 1a) and the method of identifying discrete points (Fig. 1b).

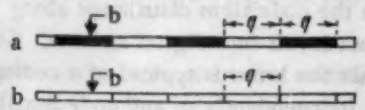

Fig. 1

For quantizing the spatial scale, physical properties connected with electrical, magnetic, optical, mechanical and other phenomena are used. Such properties can consist of conductivity and resistance, magnetization and the lack of it, transparency and opaqueness, protrusion and cavity, etc.

* They are also called: continuous-discrete converters, digitizers, etc.

The quantized scale should only be considered in conjunction with the device for distinguishing the physical features of the scale. This device must have a source of energy in the form of a generator of physical values (electrical current, magnetic or light flux, etc.) and a reproducing element (b on Fig. 1a and 1b). The latter is in the majority of cases a transducer of selected features into electrical quantities (photocell, pick-up head, etc.).

Converters of spatial quantities into a digital form can have a scale quantized with respect to another physical property, such as a time or electrical scale. In conjunction with this, other standards are chosen such as time intervals, or sources of emf or current, etc. In such cases, the measured angles or linear displacements must be first converted into time or electrical quantities.

Coding. Coding consists in establishing correspondence between each quantization level and a certain aggregate of conventional signs called a code-combination or group. The conventional signs occupy places or positions in the time of spatial scale (positional-method of digital representation). Digital elements of any one type are used for coding.

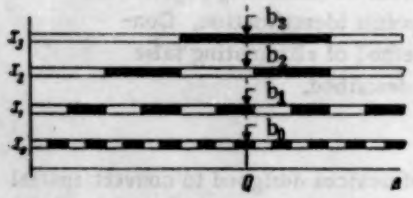

Fig. 2. Coding by means of a field of code combinations.

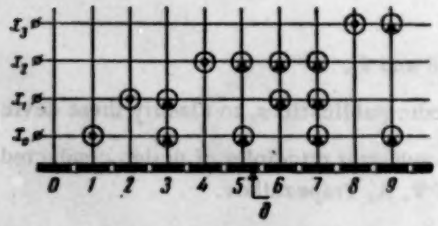

Fig. 3. Matrix method of coding.

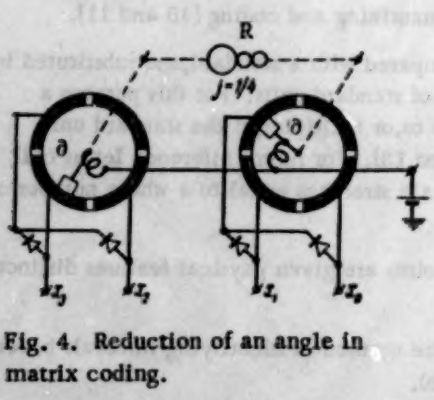

Fig. 4. Reduction of an angle in matrix coding.

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When coding by the counting method the scale quanta are converted into electrical pulses which are counted by electronic counters. The counting of the number of quanta can be made periodically (starting from zero) or additively only, when the measured value changes.

For coding by the reading-off method, code combinations for all the scale levels are established in advance, independently of the measured quantity. This can be fulfilled by two methods. The first is illustrated in Fig. 2. A number of quantized scales with identified intervals are placed parallel to axis A in such a manner that the beginning of their counts coincide. The scales have different quanta values. Figure 2 shows the case when $q_0 = q$, $q_1 = 2q$, $q_2 = 4q$, . . . , $q_{n-1} = 2q^{n-1}$. Let us call the aggregate of quantized scales, each of which serves to fill with signs the corresponding position of the code, a field of code combinations.

Coding by means of code combinations is direct. The physical properties of the code itself serve as conventional signs of the code. When the field is moved with respect to the sensing elements b_0, b_1, b_2, and b_3, the latter "read" the codes of their categories and henceforth their indications are considered as the conventional signs of the code at level Q.

The system of coordinates used for the field of code combinations depends on the character of the measured quantity. If a linear displacement is measured, the field is constructed in Cartesian coordinates, as shown in Fig. 2. When angles are measured, the field is usually constructed in the polar system of coordinates, with the code signs distributed along the radius. Coding by means of reading-off does not normally represent quanta levels in a digital manner. Codes can be assigned to levels in a completely arbitrary manner. Let us assume that the latter is typical of a coding system (or a "code system") of analog-to-digital converters. However, quite independently of any code distribution rule along the quanta levels, correspondence can be established between these levels and a set of digits by means of counting the quanta. Such a system of coding is known as the natural (or "binomial") system [11, 13]. This system can be considered as a "reference system" with respect to which all the other systems are considered

as its modifications. The cyclic system (Gray's system) is an example of nonarithmetic coding [14].

The second method of coding by means of reading-off is based on the application of connection matrices for converting scale levels to code sign combinations. Figure 3 shows a double matrix for a natural coding system. The number of lines corresponds to that of places, and the number of columns to that of quanta levels. Most of the connections must be of the rectifier type (denoted by circles with arrows). However, levels representing powers of number two can be connected to the corresponding busbars by means of normal connections (shown by circles with dots). It is interesting to note that the position of connections in the matrix corresponds exactly with the position of the identified intervals in the respective code combination fields (Fig. 2).

In matrix coding, the scale is usually quantized by identifying discrete points. It is usually made up of conducting segments with small insulated intervals.

Generating element ∂ slides over the segments providing voltage or current. The segments themselves in this case serve as receiving elements.

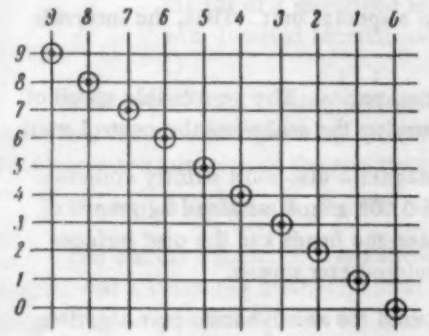

Fig. 5

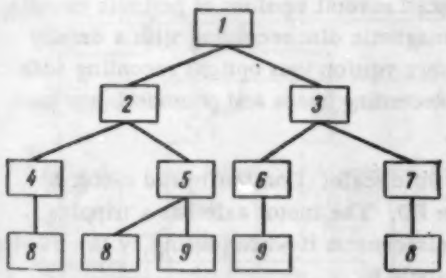

Fig. 6. 1) Digitizers of angles and linear displacements; 2) digitizers with coding by the counting method; 3) digitizers with coding by the read-off method; 4) coding by means of periodic counting; 5) coding by means of reversible counting; 6) coding by means of code combination fields; 7) matrix coding; 8) quantizing by identifying discrete points; 9) quantizing by identifying intervals between points.

The disadvantage of matrix coding consists in the great number of diodes used. This number in a binary matrix with a number of places n amounts to $(2^{n-1}-1)n$. With $n = 12$ the quantity of diodes required is 24,564. This method is, therefore, never used without a scale transformation of the measured quantity (differential method of measurement).

The scale transformation of measured quantities is widely used in analog-digital technique. In angle digitizers it amounts to the use of series reduction gears [7, 8]. Figure 4 shows schematically a matrix angle digitizer with two discs connected by a reduction gear. R with a ratio of $j = 1/4$. For four places four diodes are required with reduction, whereas without reduction 28 diodes would have been required. Very often basis C of the calculation system is made to equal the number of levels N. In this case the matrix of rectifier connections simplifies into a diagonal matrix of normal connections. Such a matrix for $C = N = 10$ is shown in Fig. 5.

A general classificational schematic of converters of spatial quantities into digital form is shown in Fig. 6.

This schematic provides quantization by means of a scale of any physical nature: spatial, time, electrical, etc.

2. Counting-Code-Converters

Devices with periodic counting. Figure 7 illustrates schematically the principle of periodic counting.

Cylinder B rotates with a constant speed ω and its axis of rotation coincides with shaft A whose angle of rotation θ is being measured. The cylinder has two paths covered by a magnetized layer. One of the paths has marks Q representing the quantizing scale recorded on it, the other has but one control mark δ.

The sensitive element consists of three heads G_1, G_2 and G_3. Heads G_1 and G_2 are stationary and placed in the plane which serves as zero reference for counting angle θ. Head G_3 is fixed to axis A.

When the cylinder is rotating, head G_1 generates pulses which are fed to the counter through switch K. The switch is controlled by heads G_2 and G_3. The switch is connected when the control mark passes over head G_2 and disconnected when the position of the mark coincides with head G_3. Thus, the number of pulses which pass through the switch over one rotation period are proportional to angle θ rounded off to the nearest quantization level. The error of approximation does not exceed the angle between two adjacent marks.

The periodic counting method was adopted in the design of the angle digitizer of the firm "ERA" (USA) [15].

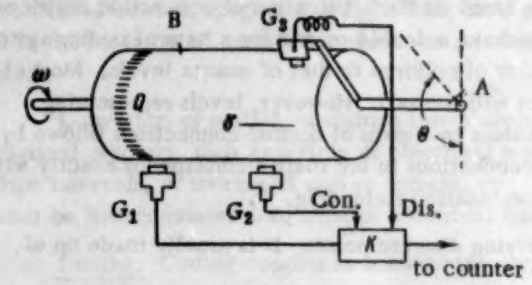

Fig. 7. Periodic count converter schematic.

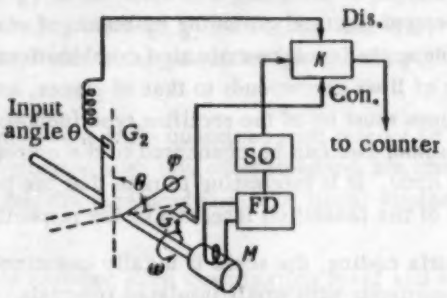

Fig. 8

This firm's digitizer is designed to measure angles up to 360° with an accuracy of ± 0.03°. An interesting design peculiarity is the frequency multiplication at head G_1. The scale has only 500 marks along the circumference. At a speed of 3600 rpm 30 kc is produced and multiplied by 8 in a special unit. Thus, the intervals between marks are indirectly divided by 8.

The periodicity of measurements is attained by external 20 cps starting pulses. The permissible speed of shaft A rotation is 120 rpm. Discs with magnetized layers are used for carrying the scale and the control mark.

In order to ensure the required accuracy the axis of rotation of the magnetic disc must strictly coincide with that of the input shaft. The centering of the axis with an accuracy of 0.005 mm is attained by means of special bearings. The airgaps between the heads and the disc surfaces (0.025 to 0.1 mm) are adjusted by micrometer screws.

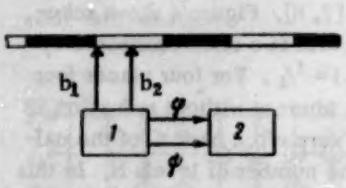

Fig. 9. Reversible counting converter circuit. 1) Direction of movement discriminator; 2) reversible counter.

The shaft angle digitizer was used for aerodynamic investigations. "Austin and Co" (USA) have improved this device and developed its miniature version [16].

"Jacobs and Co" have produced several versions of periodic counting converters [8]. One of them uses magnetic disc recording with a density of 10 to 12 pulses per 1 mm. Another version uses optical recording with a density of 80 pulses per 1 mm. Recording heads and photocells are used respectively as sensitive elements.

Figure 8 shows a shaft angle digitizer circuit with quantizing on a time scale. Constant speed motor M is fed from a standard frequency oscillator SO through a frequency divider FD. The motor axle has a tripping lever Φ which passing by the stationary head G_1 connects switch K, and disconnects it when passing by the moving head G_2. The number of pulses through the switch is proportional to the angle θ.

The essential difference between this and the circuit described above consists in the first place in the lack of a spatial quantizing scale in this arrangement and in the second place in the requirement of a strictly constant speed of rotation of motor, since it determines the linearity of the angle scanning.

Devices with reversible counting. Figure 9 shows a block schematic of a reversible counting converter. The scale with identified intervals has two sensing elements b_1 and b_2, which are displaced with respect to each other by $^1/_2$ q. The peculiarity of the circuit consists in the element discriminating the direction of the scale movement. When the scale moves from right to left the discriminator sends pulses into channel φ and the counter adds them to the previously counted pulses; when it moves from left to right the pulses are sent into channel ψ and the counter subtracts.

The logical structure of the discriminator depends on the quantizing method. With an interval discriminating method the simplest logical circuit of the combination type can be used (Fig. 10). Voltages e_1 and $\bar{e}_1$ correspond to the direct and reversed codes of trigger T_1. These voltages are differentiated by units α. The pulses thus obtained are fed to the "and" gates which only let through positive pulses when a positive voltage e_2 is present, corresponding to the direct code of trigger T_2.

Figure 11 shows the time operation diagrams of the circuit when the scale is moving from right to left (a) and from left to right (b).

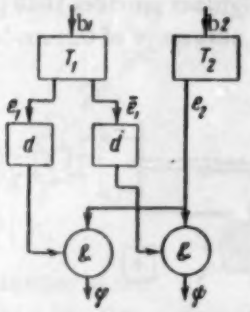

Fig. 10. Direction of movement discriminating logical circuit of a quantized scale with interval identification.

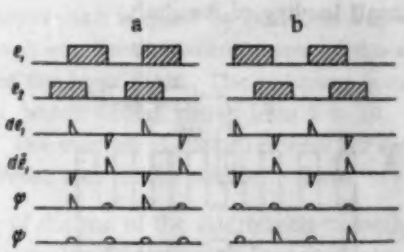

Fig. 11. Time diagrams of Fig. 10 — circuit operation.

With point identification quantizing, the logical structure of the discriminator differs from the above. In that case, the number of sensing elements must not be less than three and the logical structure of the circuit be based on discrimination of the sequence of pulses from three elements [17, 18].

The logical schematic for this case is given in Fig. 12.

The circuit consists of a synchronizing arrangement at gates a, b and c which control triggers T_a, T_b, and T_c and a switching arrangement at gates a_1, a_2; b_1, b_2 and c_1, c_2. In normal condition of the circuit all the triggers are in position 0, with the exception of one which is in position 1. Let the scale be moving from right to left with mark Q_1 having passed element b_b. This means that at that moment trigger T_b is in position 1. As soon as the mark approaches element b_a gate a will open and the next synchronizing pulse of I_s will pass through it. This pulse will throw trigger T_a into position 1, pass through gate b_2 into channel φ and operate trigger T_b into position 0. The last operation is for the purpose of blocking; it prevents the next pulse of I_s from passing into channel φ. Thus, for each Q_1 mark only one synchronizing pulse is transmitted. If the scale continues to move in the same direction the next pulse in channel φ will be received through gates c and a_2, etc. Let the scale change its direction of movement after the mark Q_1 has passed element b_a. The next pulse will only appear when mark Q_1 approaches element b_b. This pulse will pass through gate a_1 into channel φ.

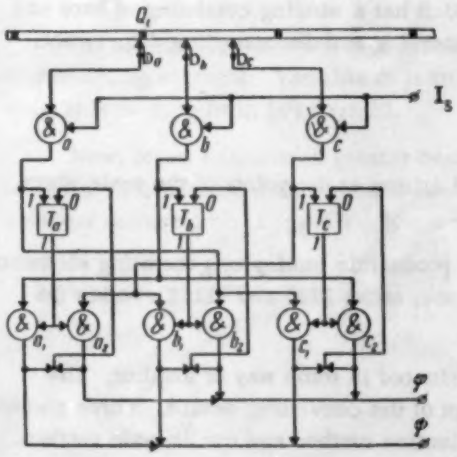

Fig. 12. Logical circuit for discriminating the movement of a quantized scale with discrete point identification.

A commercial model of a shaft position digitizer with reversible counting was produced by the "Telecomputer Corporation" (USA) [19]. The scale is produced by means of electromagnetic induction. Let us examine Fig. 13. Conductor a consisting of U-shaped sections is fed by a high-frequency current. If turn b is moved along the conductor a, an emf will be induced in the turn and it will periodically decrease to zero and change its phase by 180°. Thus, the conductor can be used for quantizing a linear scale with each section corresponding to one quantum. If the conductor is formed into a loop it will provide the possibility of quantizing angular scales within the limits of 360°.

Figure 14 shows in a simplified manner the circuit design of a digitizer. A thin celluloid disc 2 is firmly fixed on shaft 1 whose angle of rotation is to be measured. This disc has a ring with U-shaped sections imprinted on it by a photographic method. The sensing element is also made in the shape of disc 3, which has two similar concentric rings with their sections displaced by an angle $\frac{1}{2}$ q with respect to each other. This disc is stationary and is placed coaxially and parallel with the moving disc.

The ac current from oscillator 4 is induced on the moving disc through winding 5 whose axis is parallel to the axis of rotation. The frequency of the current is 1.6 Mc. When the axis is rotated the emf induced in the stationary disc is modulated by the rotating disc at a frequency proportional to the speed of rotation. This emf is amplified and fed to the direction of movement discriminator 6. The digitizer provides 1000 pulses per revolution. The permissible speed of rotation of the shaft is 1800 rpm. The advantage of this model consists in the relatively small loading of the shaft.

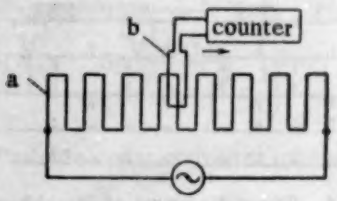

Fig. 13. Induction method of quantizing.

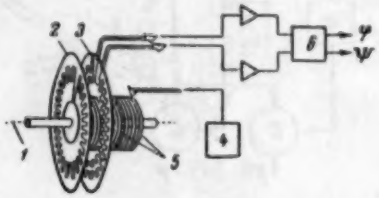

Fig. 14. Induction quantization converter circuit.

Reversible counting is widely used in systems of program control of lathes with the program set in the form of coordinate increments.

The British firm "Ferranti Ltd." has developed a precision device which uses a diffraction grating as a quantization scale [20].

Reversible counting was used in a milling machine control system developed by the MIT (USA) [17]. The quantizing part of the digitizer consists of a disc with conducting segments separated by insulated spacings. Three contact brushes run over the segments. All the conducting segments are connected to each other and they receive pulses through an additional brush. When the disc is rotated these pulses are distributed sequentially among the three channels and are fed to a reversible counter through a sequence discriminator (Fig. 12).

The "Helipot Corporation" (USA) [21] reports on the development of a shaft position-digitizer with a reversible counter. The angle is quantized by means of a ring rheostat which has a winding consisting of bare and insulated wires. A contact brush slides over the winding alternately connecting and disconnecting the current. The number of pulses for a complete revolution is equal to 2000.

3. Read-Off Coding Converters

A characteristic of read-off coding is the possibility of false codes arising at the points of the scale where the signs of the code change in more than one order.

False codes arise due to the finite dimensions of any physically producible sensing or generating elements as the result of which the codes of adjacent levels overlap. If, for instance, codes 1000 and 01111 overlap the error will amount to 50% of the full scale corresponding to code 11111.

In practical designs the possibility of producing false codes is eliminated in some way or another. The method of eliminating false codes influences to a great extent the design of the converting device. Three methods are the most commonly used: the discretization method, the logical selection method and cyclic code method.

Devices using the discretization method. By discretization of analog quantities we shall understand the substitution of the continuous function representing the analog quantity, by a discrete function. The aim of this substitution is the elimination of the undesirable parts of the function which correspond to the changes from one quantization order to another.

An angle can be discretized, for instance by a locating star wheel (Fig. 15). Locating star wheel 2 is firmly fixed on axle 1, its number of cogs being equal to that of quantization levels. The reading-off is made periodical by means of switch 3. The operation of the switch energizes solenoid 4 which fixes the position of the wheel at the nearest quantization level. Axle 1 is connected to the input shaft by spring coupling 5 [22].

This principle is used by the "Fischer and Porter Co." [23]. The device consists of a number of cylinders connected to each other by means of reduction gears; the cylindrical surfaces containing the code combination

fields. The code signs are represented by juts and notches. Each cylinder is provided with contact springs which control external electrical circuits. The contact springs are periodically pressed against the cylinders by means of solenoids. If one of the springs falls on a jut, it closes the corresponding electrical circuit, if it falls on a notch the circuit remains open. The springs identify the position of cylinders only at certain levels avoiding false codes due to overlapping. The connection to the input shaft is made by means of a special coupling free from hysteresis, which ensures the measurement of the angle without disturbing the rotation of the input shaft. The accuracy is controlled by the number of cylinders, which can be varied from 2 to 10. The loading moment is 14.4 g-cm. The number of measurements per second does not exceed three. The external view of the device is shown in [8].

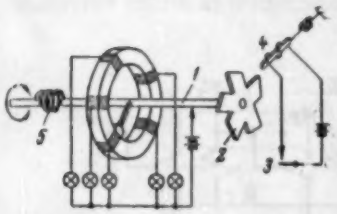

Fig. 15. Discretization of the angle by means of a locating star wheel.

Another example of the use of the discretization method is provided by the "Dayton Instruments, Inc." digitizer. Its peculiarity consists in the input shaft being connected to the cylinder but to a contact frame. The closing of the contacts only occurs at the time of identification which prevents excessive wear. The digitizer is designed to work with perforators, tabulators, etc. [24].

The discretization method is also used in devices of the cathode-ray tube type. A well-known example is provided by the coding tube with the so-called "quantizing grid" [25].

Devices with a logical selection of sensing elements. The physical properties of a quantized scale can be identified with the truth values of variables in the logical calculus. Let us assume that the logical variable x represents the output characteristic of the sensing element and becomes true ($x = 1$) when a "black" portion of the scale is opposite the sensing element, (Fig. 1) and it becomes false ($x = 0$) when a "white" portion is opposite the element.

Denoting by $d(x)$ the transition of the true value of the variable to a false one and by $D(x)$ the variation of the truthfulness values of the variable [26],* we have

$$D(x) = d(x) \vee d(\bar{x})$$

Let us also introduce a logical variable σ, which shows the direction of the scale movement with respect to the sensing element. Variable σ is true ($\sigma = 1$) when the scale is moving from right to left and false ($\sigma = 0$) when it is moving from left to right.

Now, let us examine in greater detail the character of variations in the truthfulness values of logical variables in quantized scales of code combination fields for a natural coding system (Fig. 2). For variables of adjacent orders x_k and x_{k+1} $(0 \leq k \leq n-2)$ the following logical formulas hold:

$$d(x_k) \& \sigma \to D(x_{k+1}), \tag{1}$$

$$d(x_k) \& \bar{\sigma} \to \bar{D}(x_{k+1}), \tag{2}$$

$$d(\bar{x}_k) \& \sigma \to \bar{D}(x_{k+1}), \tag{3}$$

$$d(\bar{x}_k) \& \bar{\sigma} \to D(x_{k+1}). \tag{4}$$

From these formulas it follows that when $x_k = 1$, changes in the truthfulness values of x_{k+1} do not occur rapidly if the scale of the $(k+1)$-th order is moved from left to right ($\sigma = 0$) [see formula (2)], and when $x_k = 0$ the same holds good if the scale is moved from right to left ($\sigma = 1$) [see formula (3)].

In the first instance element b_{k+1} can be moved toward the origin of the coordinates to at least the distance equalling q_k, without any change in the truth of x_{k+1} and in the second instance it can be moved from the origin with the same effect.

* The notation of logical relations is the same as that adopted in [27]: $\bar{x}$ is the negation of x, $\vee$ is the sign of disjunction, & is the sign of conjunction and $\to$ is the sign of implication.

Let us now supply each order, with the exception of the zero order, with two elements, and let us place them with respect to the position line of the zero order element, as shown in Fig. 16. Let us call elements $b_1, b_2, \ldots,$ etc. retarded, and elements $b'_1, b'_2, \ldots,$ etc., advanced. Let us make the displacement of elements with respect to the line of symmetry equal to $2^{k-2}q$.

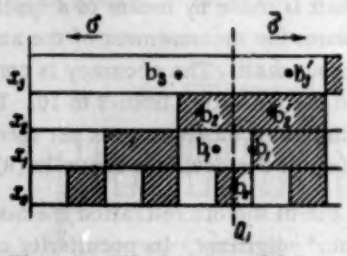

Fig. 16. Elimination of false codes by means of logical selection of sensing elements.

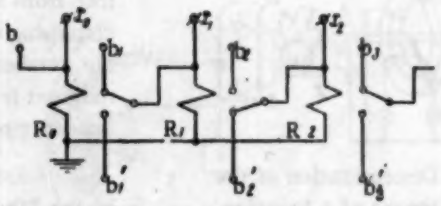

Fig. 17. A logical circuit made up of electromechanical relays.

The choice of the sensing element for the $(k+1)$-th order should be determined from the truthfulness of the logical variable of the preceding order. If $x_k = 1$, a retarded element is chosen for reading-off; if $x_k = 0$, an advanced element is chosen. It is easy to see that with such a method of reading-off no false codes will arise.

The choice of reading-off elements is made by the logical circuit. An example of such a circuit using electromechanical relays is shown in Fig. 17. The condition of the circuit corresponds to the code shown in Fig. 16. The relays are energized when the corresponding elements get into the shaded area of the code combinations field. The method of logical choice is often called V-reading, since the position of the elements usually resembles the letter V [28, 29].

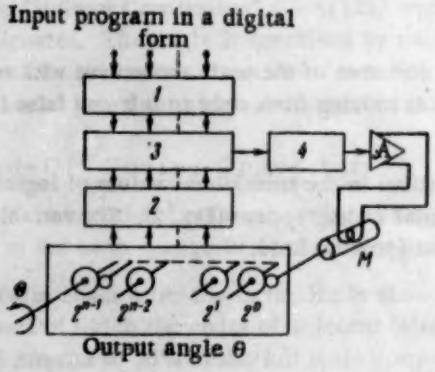

Fig. 18

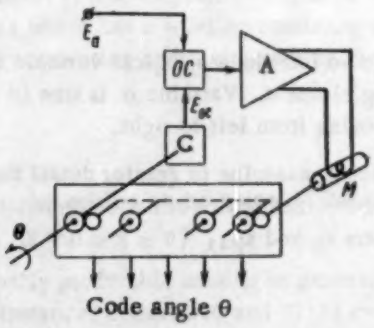

Fig. 19. Digital-recording circuit for converting an angle.

Contact brushes often serve as receiving elements. In important cases optical-type elements are used with two staggered scanning zones of the code combinations field [30].

The logical-choosing method, especially, is widely used in precision-reduction digitizers of angles when the measured angle becomes the output quantity of the subsequent system.

Figure 18 shows the block schematic of the servo-system which serves as a digital-to-analog converter [31]. The incoming-recorder code 1 is compared with the performance-recorder code 2 by means of the comparator unit 3. Mistakes in the form of digits are converted into analog voltages by unit 4. Voltage amplifier A controls the servomotor M which in turn controls angle θ by means of a series of reduction gears. The analog-to-digital conversion is accomplished either by means of a code combinations field or by matrix connections (Fig. 4). Many lathe program control systems are constructed on this basis with the program set in the form of absolute values of coordinates [32].

Figure 19 shows even a more widely used circuit which consists entirely of analog elements. The input voltage E_a is compared with the feedback voltage E_{oc} obtained by converting angle θ in the C unit. The difference is amplified by amplifier A and operates servomotor M which controls angle θ through a series of reduction gears. Some automatic compensators with digital recording (printing, perforations, etc.) of such measured values as temperature, pressure, etc., are constructed on this principle [7, 33].

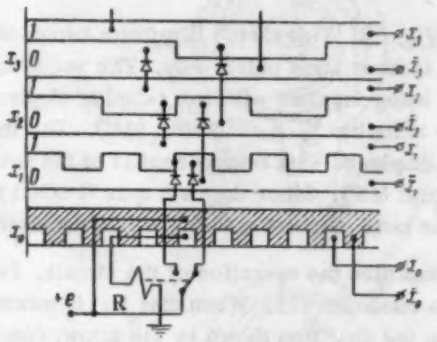

Fig. 20. Code combination field and logical choice circuit of a "Norden Ketay "digitizer."

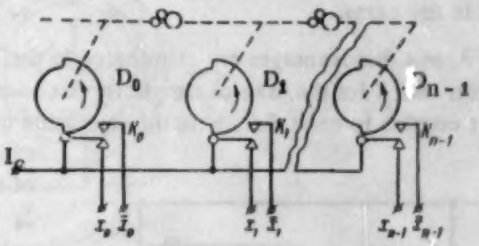

Fig. 21. Shaft-angle cam digitizer.

A large number of shaft-angle digitizers have been developed for such systems. The series of instruments developed by the firm "Librascope Inc." serves as a good example [8, 34]. A typical instrument of this series consists of discs connected by means of reduction gears. The code combinations field is made up of conducting and insulated segments. Contact brushes serve as sensing elements. The instrument is designed for a maximum shaft rotation speed of 120 rpm. At this speed the contacts can withstand $5 \cdot 10^8$ operations. The current drain is 2 ma per brush, the accuracy 7, 13, 17, and 19 binary orders. Some of the models use a binary-decimal code. The appearance of the sets and their logical choice circuit are shown in [8].

The firm "Norden Ketay" has developed a digitizer in which the logical choice of the read-off elements is achieved by a special construction of the code combinations field (Fig. 20). The column of each order with the exception of the lowest order consists of two parts the "unit" and the "zero" part. These parts are made of a conducting material and separated by insulated spaces. All the columns, with the exception of the lowest-order one, have four contact brushes. The extreme-left brushes slide over the unit parts of their columns and extreme-right ones over the zero parts. The two central brushes serve for selection. They perform the functions of the retarded and advanced elements and are therefore connected respectively to the unit and zero parts of the preceding orders. Gates prevent the unit and zero parts from shorting when the selecting brushes of the succeeding order are on the same part. The column of the lowest order is made of conducting and insulated strips. It operates relay R which provides an electrical potential to the retarded or advanced brushes of the next order column according to the condition of the lower order. The appearance of the set and a part of its coding disc are shown in [8]. The number of binary orders is equal to 13 (with the use of reduction gears). The recommended speed of shaft rotation is 200 rpm.

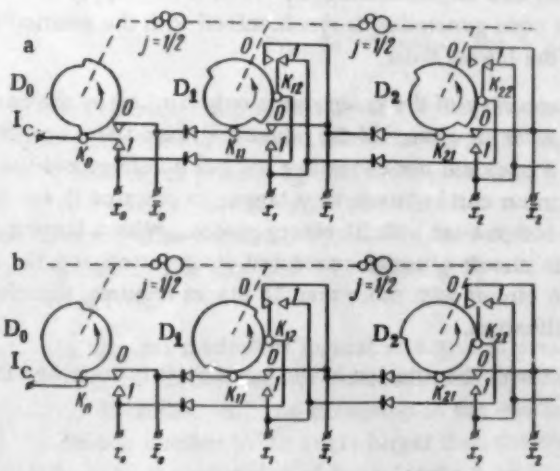

Fig. 22

The positive characteristic of this converter consists in the possibility of obtaining both direct and reversible codes [29, 35].

The logical selection method is also applied in devices which use mechanical switches as sensing elements. Such a device is described in [36].

Its basic diagram is shown in Fig. 21. The device uses cam type coding discs. The sensing elements are made of two-position switches which are periodically tested with pulses I_c. Adjacent discs are interconnected by

means of reduction gears of ratio $j = 1/2$. Two disadvantages of this circuit are obvious: 1) when disc D_0 is rotated through 180° the digitizer should pass from condition 111...1 to condition 000 ...00; with 12 orders, however, the last disc D_{n-1} will rotate only through $1/4096$ of a revolution, which will not bring switch K_{n-1} out of an indefinite condition; 2) when the condition is changed from 111...11 to 000...00 all the switches must change simultaneously. This is impossible, due to the different speed of disc rotation and manufacturing errors in the cams.

Above disadvantages are eliminated in the circuit of Fig. 22. This sketch illustrates two conditions of the digitizer which for the sake of simplicity has been assumed to have three orders only. The peculiarity of the circuit consists in each disc, with the exception of disc D_0, being supplied with two switches displaced with respect to each other by $1/2$ a quantum. (90°). The initial position of the disc is displaced with respect to that of the lower order by $1/4$ of a quantum (45°). Since the cam span is equal to 180° the switches of the same disc are never operated simultaneously.

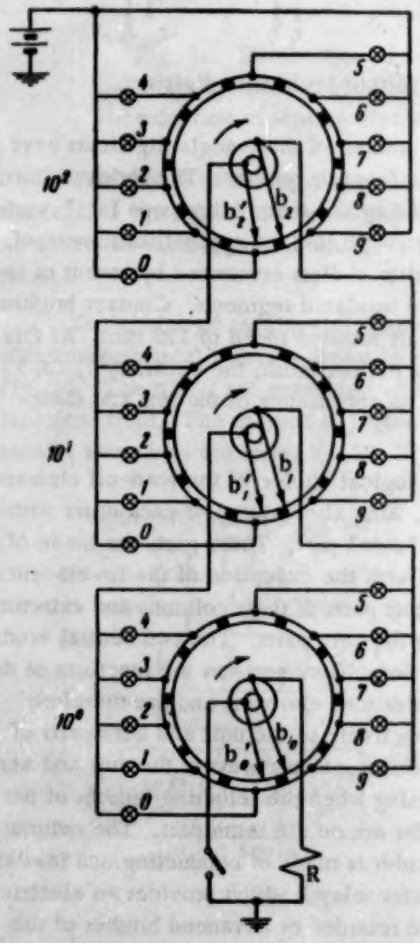

Fig. 23. "Coleman Eng. Co." shaft-angle digitizer.

Let us examine the operation of the circuit. Figure 22a corresponds to condition 111. When disc D_0 is rotated through a small angle in the direction shown by the arrow, condition 111 should change to 000. This is attained immediately on switch K_0 being thrown from position 1 to 0, independently of the position of switches in all the remaining orders. When the disc is rotated through 180° the digitizer will approach the end of condition 000 (Fig. 22b). The position of switch K_{22} will be indeterminate. This will not, however, affect the read-off results since at that time switch K_{21} will be effective and not K_{22}. The voltage supply to that switch is ensured by the position of the preceding disc.

In analyzing the circuit it is easy to note that if any switch (with the exception of K_0) is in an indeterminate position, both switches of the preceding order disc are either in the position 1 or 0 so that no read-off pulse is supplied to that switch.

An indeterminate reading could arise when the read-off pulse is supplied at the instant of switch K_0 operation. This uncertainty can also be eliminated, however, if the supply of the pulse or the code generation is synchronized with the position of the disc of the lowest order.

The accuracy of the converter is only limited by the backlash of the reducing gears. If the converter is used as a counter of revolutions a backlash not exceeding 45° has no effect and the number of orders can be made very large. In practice it was found possible to design a set with 31 binary places. With a large number of places the circuit of series connected diodes attenuates the read-off pulse. A circuit with more than 15 places requires, therefore, pulse amplification.

The method of synchronizing the reading of the code with the rotation of the input shaft is described in [37].

Let us now consider examples of matrix coding.

Figure 23 illustrates the principle of operation of the "Coleman Eng. Co." digitizer [38, 39]. The digitizer is assembled from discs each with 10 segments (C = N = 10). The segments are connected to indicators denoted on the drawing by numbers. Each disc has two contact brushes separated by a distance greater than the insulation spacing between the discs. The electrical circuit of brush b_0 is completed through relay R and of brush b'_0 through the normally closed contact of the same relay. Thus, brush b'_0 becomes operative when brush b_0 is between two segments.

The drawing shows the beginning of condition 000 when brushes b'_0, b'_1, b'_2, etc., are operative. After the first half-turn of brushes b_0 and b'_0 brush b_1 becomes operative; after the second half-turn it is brush b'_1 again. The changing of operative brushes in the succeeding orders is carried out in a similar manner.

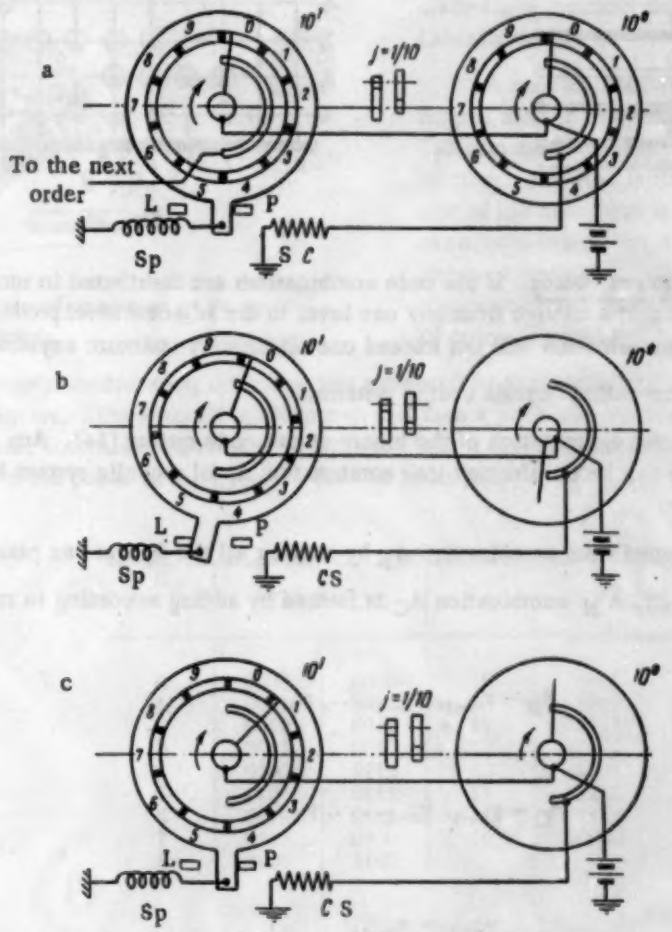

Fig. 24. "Genisco Inc." shaft-angle digitizer.

The characteristic of this device is the extremely small loading moment amounting to 0.36 g-cm. It was possible to obtain such a small moment due to the separation of the contact brushes from the segments. The contact brushes are pressed against the segments only at the instant of reading-off the code. This is attained by means of a solenoid whose excitation circuit is either independent or related to the rotation of the lowest-order-brushes shaft. The permissible rotation speed of this shaft is 18,000 rpm. Standard models are designed for 3, 4, and 5 decades. The size of a 4 decade digitizer is 125 × 125 × 28 mm, and the weight 300 g. Its appearance is shown in [39].

An original method of logical selection is described by the firm "Genisco Inc." [40]. The device consists of a number of contact brushes interconnected by reduction gears, each brush sliding over a segmented disc (Fig. 24). Each disc with the exception of the one of the lowest order can take up two positions displaced with respect to one another by an angle larger than the distance between the segments. The position of the disc is determined by the condition of the solenoid S whose electrical circuit is closed through the segment of the preceding order disc which has a span of 180°. After the first half-turn of the lowest-order-brush the solenoid is de-energized and the disc of the next higher order (10^1) is rotated to position L under the action of the spring, Sp. (Fig. 24b). After the next half-turn the solenoid is energized and returns the disc to its initial position P (Fig. 24c). The contact brush in the meantime "jumps" across the indefinite position between the contacts and takes up its position on the next segment. Thus, in this case, the method of logical selection does not lead to the doubling of the number of contact brushes but is catered for by doubling the number of positions for the quantizing scales.

The accuracy of the instrument can be varied by increasing or decreasing the number of decades. The loading moment is about 100 g-cm. This device is recommended by the firm for measuring shaft rotation angles of lathes, servomotors, and as revolution counters.

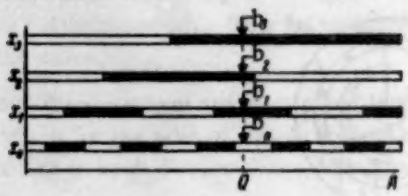

Fig. 25

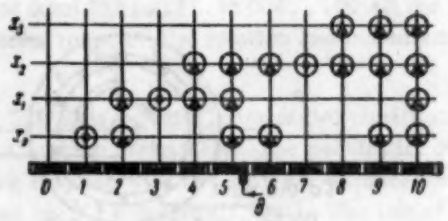

Fig. 26

Devices using a cyclic system of coding. If the code combinations are distributed in such a manner over the levels of the quantizing scale that a change from any one level to the adjacent level produces a change in signs only in one order, the error of transformation will not exceed one elementary quantum anywhere.

This is the property of the so-called "cyclic coding systems."

We shall limit ourselves to the examination of the binary cyclic code system [14]. Any combination A_N of the natural binary code system can be transformed into combination A_C of a cyclic system by applying the following rule.

1. Combination A'_N is formed from combination A_N by moving all the figures one place to the right.

2. From combinations A_N and A'_N combination A_C is formed by adding according to mode figures of corresponding places, i.e., if

$$A_N \sim x_{n-1}, x_{n-2}, \ldots, x_0 \tag{5}$$

and

$$A_C \sim y_{n-1}, y_{n-2}, \ldots, y_0, \tag{6}$$

then

$$y_{n-1} = x_{n-1}, \tag{7}$$

$$y_k = x_k \& \bar{x}_{k+1} \vee \bar{x}_k \& x_{k+1} \qquad (k = 0, 1, 2, \ldots, n-2). \tag{8}$$

It follows from here that the transformation of a natural code system into a cyclic one can be accomplished by means of a half adder [41].

This transformation has a very simple geometrical interpretation. For a natural code system (Fig. 2) the length of the identified (black) and the unidentified (white) intervals is equal to $2^k q$ where k is the number of the order. The shift with respect to the origin of the first identified intervals counting from the origin is also equal to $2^k q$.

For a cyclic system the length of both types of intervals is doubled, i.e., becomes equal to $2^{k+1} q$, but the shift of the first identified intervals remains the same (i.e., $2^k q$).

The cyclic code combinations field is shown in Fig. 25.

Figure 26 shows an example of matrix coding by means of a cyclic code system.

The table gives a digital notation of a natural series of numbers (I) in the binary natural code (II) and in the binary cyclic code (III).

At present the cyclic system of coding is widely used in the analog-digital technique. This system is used in almost all the shaft-angle digitizers with optical sensing elements. It is generally recognized that the

greatest measurement resolution of spatial quantities is attained by optical means. This method forms the basis of precision measuring instruments [12].

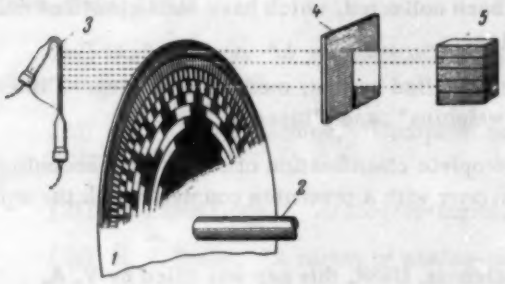

Fig. 27. Principle of operation of an optical-shaft angle digitizer.

Figure 27 illustrates the principle of operation of a shaft-angle digitizer developed by the "Bell Telephone Laboratories" [5].

A code combinations field in the shape of concentric rings is affixed to a glass disc 1, of 6.5 mm thickness and 140 mm diameter, the rings having transparent and opaque sections. The disc is fixed to the input shaft 2. On one side of the disc there is a powerful source of light 3, and on the other side a screen, 4 with a 0.025-mm slit and a set of photoconductive cells 5. The light or darkness of the photoconductive cells forms the code signs. The number of orders is equal to 18.

The greatest manufacturing difficulty was presented by the coding disc. The codes were affixed by a photographic process. First a negative was drawn and then a print was obtained by contact printing. For plotting the code a special automatic device was constructed with a dividing mechanism and a control system. The automatic drawing of the codes on the negative took 20 hours [5].

I	II	III	I	II	III
0	0000	0000	9	1001	1001
1	0001	0001	10	1010	1111
2	0010	0011	11	1011	1110
3	0011	0010	12	1100	1010
4	0100	0110	13	1101	1011
5	0101	0111	14	1110	1001
6	0110	0101	15	1111	1001
7	0111	0100	16	10000	11000
8	1000	1100			

A similar device was being developed by the Technical Laboratory of the U.S. Signal Corps [12]. This device had a disc diameter of 237 mm and its number of orders was 15.

Recently articles appeared in the Technical Literature on the development of instruments of a similar type. "The Baldwin Piano Co." has produced a digitizer with a division of 360° into 65,536 parts (16 binary places), i.e., with a quantum of 10 minutes of the arc [42]. According to the firm's data the coding disc segments have been made with an accuracy of 0.0008 %. A small neon tube fed from the 60 cps supply is used as the source of excitation. Thus, the frequency of the shaft position measurements is equal to 60 cps. The sensing element consists of phototransistors. The width of the slit is 0.005 mm. The disc has also a transparent region and the phototransistor corresponding to it serves to compensate the variations in the luminous intensity of the neon tube. Each transistor has an amplifier with an internal resistance of less than 1000 ohms at whose output a signal of 5 to 7 v is produced.

The "Electronic Corporation of America" reports on the development of an optical shaft-angle digitizer [43]. This firm has developed a rather compact device with the following technical data: elementary quantum of less than 3" of the arc, frequency of measurement up to 100 cps, effective moment of 14.3 g-cm, weight 1.72 kg, and the permissible shaft-speed of rotation 150 rpm. A gas discharge tube of the "pencil" type with a high luminosity is used as a source of excitation. The output of each photoconductor is connected to a built-in transistor-amplifier which raises the signal level to 1.5 to 3 v across a 3000 ohm output resistance.

The valuable advantage of optical shaft-angle digitizers is the absence of mechanical contacts. The disadvantages consist in the difficulty of manufacture and more exacting operating conditions. These devices are usually employed in ground radiolocation sets and instruments for the control of anti-aircraft artillery.

This far-from-complete review of analog-to-digital converters shows that at present these devices have developed into an independent group of automatic instruments. The principle of operation of these instruments is based on two stages: quantizing and coding.

Classification of the analog-to-digital converters has been carried out in several papers for instance [11, 22] etc. In paper [22] a number of examples of converters has been collected, which have been classified rather by their design characteristics than by their principle of operation.

In the work of Lippel [11, 13] for the first time they were classified by their methods of coding. The converters were classified in four groups: "digital," "reading," "weighing" and "inverted."

The drawback of article [11] is the failure to provide a complete classification of the devices according to above groups. This refers for instance to the shaft-angle converter with a reversible counter, which the author called a "misfit."

In the work carried out by the IARC of the Academy of Sciences, USSR, this gap was filled by V. A. Trapeznikov. He proposed to classify this type of converter as a device with a spatial quantizing scale and digital coding.

In the present article it was also considered advisable to classify converters according to the method of scale quantization. The differentiation of two methods of quantization — by identifying intervals and by identifying discrete points — provides the possibility of characterizing in a single-valued manner the read-off method of coding. Above methods of quantization determine more precisely the logical structure of the direction discriminators used in devices with reversible counting.

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